foreword to the MONTHLY[1] in 1913 and the tentative platform[2] of the Association in 1916: "It is not the province of the MONTHLY to enter the field of general pedagogy, nor will the MONTHLY entertain discussions that are concerned with research in general pedagogy, or that deal with new theories of pedagogy, however important these contributions may seem. . . . What we desire is to inspire, not a discussion along these lines, but rather a discussion of definite mathematical problems. . . . Just as research will be held to be within the province of the Association only if the word is given a broad interpretation, it may be said also that the discussions which this Association will foster may be termed pedagogical only if the word is used in a much broader sense than is common."

To sum up the matter now, after five years of experience, the activities of the Association bearing upon the teaching of mathematics include at least the following: (1) the presentation, discussion, and publication of papers calculated to stimulate general mathematical interest and professional esprit de corps, or of papers calculated to stimulate thoughtful consideration of possible new courses or reconsideration of the content and form of old courses; (2) the investigations and reports of committees such as the Committee on Libraries which recommended minimum lists of books for reading in the various college years, the National Committee on Mathematical Requirements, which is now engaged in a complete reconsideration of the curriculum content from the elementary school to the college, and the Committee on Mathematical Dictionary which desires to see in the hands of every teacher and student of mathematics a source of information giving in clear, compact, and scientific form an explanatory definition of every mathematical term which is likely to be found in their reading or study.

It should be added in this connection that the foregoing remarks concerning technical pedagogical matters refer to the formal meetings of the Association and formal papers in the MONTHLY. The numerous sections of the Association provide sufficiently small units to give full opportunity for informal discussions of any character and to any extent that may be deemed desirable by the members concerned; and the department of Questions and Discussions in the MONTHLY provides an open forum for free discussion in brief form of any questions deemed by the editors to be of interest and value.

Turning now to the broader retrospect of American Mathematical activities, we find the field well classified by Professor Fiske in his presidential address in 1904, namely, including the colonial period, up to the founding of Johns Hopkins University in 1876, the period from 1876 to the nationalizing of the New York Mathematical Society in 1891, and the period from 1891 to 1904. Our retrospect need not extend back of this latter date, except that it will be convenient to consider here the entire twenty year period from 1900 to 1920. The mathematical development during this period may be gauged pretty accurately on the scientific side by the activities of the American Mathematical Society and by the rapid expansion of the American colleges and universities; and on the teaching side by the activities of the secondary associations and more recently of the Mathe-

[1] Vol. 20, *1913*, 4. [2] Vol. 21, *1916*, 33.

matical Association of America, and by the rapid expansion of the American high schools, both within their own original boundaries and also downward to include the junior high schools and upward to include the junior colleges.

During this twenty-year period the Society has held six of its eight colloquia[1] at which series of from four to six lectures each were delivered by Professors Oskar Bolza, E. W. Brown, E. B. Van Vleck, H. S. White, F. S. Woods, E. H. Moore, Max Mason, E. J. Wilczynski, G. A. Bliss, Edward Kasner, L. E. Dickson, W. F. Osgood, Oswald Veblen, and G. C. Evans. Several of these colloquia lectures were subsequently published in book form. The attendance upon these lectures has averaged 48, the highest being 69 at Cambridge in 1916.[2]

The Society has held during this period twenty summer meetings, with an average attendance of 48 and a maximum of 80 at Ann Arbor in 1919, at which 606 scientific papers were read; twenty February meetings and twenty October meetings in New York, with an average attendance of 34 at which a total of 471 papers were read; eighteen December meetings in the east, chiefly in New York, with an average attendance of 68 and a maximum of 131; and twenty December meetings in the middle West, chiefly in Chicago, with an average attendance of 43 and a maximum of 84, at which 514 and 407 papers respectively were read; nineteen April meetings in New York with an average attendance of 49 and a maximum of 82; twenty April meetings in Chicago with an average attendance of 41 and a maximum of 69, at which 422 and 442 papers respectively were read; thirty-five meetings of the San Francisco Section with an average attendance of 14 at which 299 papers were read; and twelve meetings of the Southwestern Section with an average attendance of 18, at which 212 papers were read. The total number of meetings held by the Society and its sections during this period is thus 184 and the number of scientific papers read is 3373. Of these papers 2155 had been published when the annual list for 1919 was made up.

While this conspectus of meetings of the Society manifestly gives only a quantitative measure of some of the scientific activity of American mathematicians, it nevertheless provides a fairly definite indication of progress in research work. The lists of papers read before the Society and subsequently published show an average annual output for the twenty years of 108 papers. These papers and numerous others read before other scientific bodies such as local and state academies of science, the National Academy of Science, national and international congresses, together with monographs and treatises published by educational foundations, by universities, or by commercial publishers, together constitute a scientific output in mathematics which not only makes a favorable showing for the past but also augurs well for the future.

Just as the attendance upon meetings and the output of scientific papers provide a quantitative measure of the contribution to mathematical progress made through the American Mathematical Society, so the number of doctorates

[1] Not including the Evanston Colloquium in 1893 at which Professor Felix Klein delivered a series of lectures.

[2] The ninth colloquium was since held at Chicago with an attendance of 88.

in mathematics, with their rate of increase and distribution, furnishes a certain quantitative measure of the contribution made by the universities. From 1898 to 1908 the average yearly number of doctorates in mathematics was about 12, while from 1908 to 1918 the average more than doubled. In 1900 only five universities granted the doctor's degree in mathematics, and to a total of eleven candidates. In 1916, thirteen universities granted the degree to a total of 35 candidates, the largest number in any year up to that time. While it is true that more than three-fourths of all the doctorates in mathematics from 1900 to 1920 have been given by eight universities, namely, Chicago 93, Johns Hopkins 39, Yale 38, Harvard 37, Cornell 31, Columbia 30, Pennsylvania 21, and Clark 16, a total of 305, nevertheless, twenty-nine different universities have contributed to the grand total of 406 mathematical doctorates during this period. It was approximately in 1910 that several other institutions began seriously to make their contributions to such doctorates. These are notably, Illinois with a total to date of 17, California with 15, Princeton with 15, and Michigan with 10. Other universities with more than one doctorate in this period are: Wisconsin 7, Syracuse 6, Indiana 5, Bryn Mawr 5, Virginia 4, Catholic University 3, Boston, Missouri, and Kansas, each 2.

We thus have had eight universities steadily and consistently throughout the twenty years contributing highly trained men and women to the mathematical faculties of our institutions, and four other universities doing likewise during the past ten years, while still others are showing signs of similar activity. The significance of these figures is not so much in the actual output, for this is small indeed compared to the needs, as in the portent for the future. Not one of the twelve universities just mentioned has reached anything like its limit of capacity for turning out trained men, while at least another dozen may be added to the productive list in the next decade, stimulated, as they are bound to be, by the cumulative influence of the steadily increasing body of trained mathematicians. What has been said with respect to mathematics is equally true of practically all other sciences, and in some cases with greater emphasis, so that the combined influence of the rapidly growing body of trained scientific men is certain to elevate at least a score of American universities to high rank in the production of research scholarship in the next twenty years.

Having deliberately laid primary emphasis upon the development of research scholarship in this country during the past twenty years, let us now briefly review the activities of this period with respect to the improvement of mathematical teaching. It is not too much to say that the presidential address of E. H. Moore in December, 1902, started a train of thought and action whose far-reaching influence has extended through all these years to the present time. Its first immediate effect was the expansion of the Central Association of Physics Teachers, then just organized in Chicago, into the Central Association of Science and Mathematics Teachers, an organization which has always been and is now at the very front in the promulgation of progressive ideas on the teaching of secondary mathematics. Similar organizations at once followed in 1903 in New

England and in the Middle States and Maryland, and a rapid succession of state and local organizations ensued, especially in the Middle West, and reaching to the Rocky Mountains and the Pacific Coast. There seemed to be a spontaneous awakening and a universal desire to reconsider the very fundamentals of procedure in the teaching of mathematics. There followed at varying periods committee investigations[1] and reports on geometry, on algebra, on combined mathematics, on mathematics for students of engineering, and on all phases of facts concerning the teaching of mathematics in this country gathered and summarized by the fourteen American committees under the International Commission on the Teaching of Mathematics.

All of these activities are in a sense now centered in the National Committee on Mathematical Requirements which was organized by the Mathematical Association of America as one of its earliest official actions, and which later became still more truly representative of national interests when it was enlarged to include members from the three great secondary associations of mathematics teachers and from the southwestern and Pacific coast sections. This Committee is unique in several respects. It is adequately financed by the General Education Board which, after careful investigation, considered the work of the Committee of sufficient national importance to warrant a liberal appropriation of funds for its use last year and a largely increased appropriation for a second year. The committee is able, therefore, to hold periodic meetings with all members present; it can command the services of a chairman and vice-chairman who give their whole time and attention to the weighty questions with which it is concerned; it can keep in touch with all the organizations of mathematics teachers in the country, not merely by correspondence, but especially by personal contact in their meetings, thus gathering at first hand the results of cooperative activity from all sources. The scope of this committee's activity is unlimited. While it is at present engaged in consideration of the secondary field, it is bound eventually to reach down into the junior high school and elementary school curricula, and to reach upward into the junior college domain. Finally, this committee enjoys the confidence of the Bureau of Education at Washington, so that its reports are published as public documents.

Never before in connection with mathematics in America has there been a committee with such an organization and such an opportunity for effective action. Possibly never before has there been greater need of just such an organized body of friends of mathematics as this committee represents. The whole high-school curriculum has been called in question by would-be reformers who would seek to justify the retention of any subject in the curriculum only in so far as its direct usefuness in the practical affairs of life can be established. Mathematics has come in for its full share of criticism, and by many has been weighed in the balance to their own satisfaction and found wanting. It is maintained by them that on the score of practical usefulness in the ordinary occupations of

[1] See an Editorial on "Incentives to mathematical activity," AMERICAN MATHEMATICAL MONTHLY, *1913*, 169–173.

life, no general brief for algebra and geometry can be defended; that on the score of training special faculties, such as memory and reasoning power, the acquirements in mathematics do not carry over in general to other fields; that on the score of general culture the claims of mathematics which have held it as a required course in the curriculum have been grossly exaggerated. Hence, they conclude that, except for those looking forward to technical courses or to teaching mathematics, algebra and geometry should be removed from the curriculum as required subjects.

It is not my purpose to discuss here the validity of any of these claims against mathematics—there has been much discussion pro and con in the public press. Suffice it to say that, while in some school systems action has been taken toward eliminating or greatly reducing the required work in mathematics, nevertheless, the public in general is not convinced of the wisdom of such action, the psychologists are not agreed among themselves as to their claims, and the conclusions drawn from systems of mental tests are far from convincing. It is doubtless true that the personal experiences of a large multitude of individuals belie the charges against mathematics and hence the thinking public stands unconvinced.

However, no one would claim that the content and method of presentation of our mathematical courses, whether in elementary school, high school, or college, are beyond improvement; in fact, probably the most merciless critics of content and method are the mathematicians themselves. In no other branch of the curriculum has there been so much thoughtful agitation for improvement. It is well that the plans for improvement are in the hands of experts in the mathematical field. This is the service which the Mathematical Association of America has rendered to the cause in establishing the National Committee on Mathematical Requirements. We have faith to believe that the present uncertain state in the elementary and secondary fields will be satisfactorily adjusted through the action and influence of this National Committee.

Meanwhile, the Association, through the MONTHLY, through its national meetings and the meetings of its ten sections, through the widening circle of undergraduate mathematical clubs, through the expanding influence of its members as they become more and more conscious of the opportunity for improvement in the collegiate field and more and more enthused with zeal for promoting the interests of mathematics both pedagogically and scientifically, will go forward vicariously fulfilling the prophetic vision which Professor Moore had in mind for the Society in 1902 when he said, in his presidential address: "Do you not feel with me that the American Mathematical Society, as the organic representative of the highest interests of mathematics in this country, should be directly related with the movement for reform? And, to this end, that the Society enlarging its membership by the introduction of a large body of the strongest teachers of mathematics in the secondary schools, should give continuous attention to the question of improvement of education in mathematics, in institutions of all grades?" Thus, what Professor Moore and many others had hoped would be the rôle of the Society, has, by mutual and satisfactory arrangements, fallen to the lot of the Association.

In conclusion, let me speak briefly of the serious economic conditions which confront the Association in common with the Society and most other scientific organizations. The great increase in the cost of printing and supplies will doubtless make necessary an advance in the annual dues of the Association by at least thirty-three and one third per cent. [an advance since authorized]. This seems a modest increase in comparison with most other kinds of expenses, but it will be a real burden for a large number of members in the colleges and universities where salary advances have not kept pace with the increased cost of living. Moreover, this increase will scarcely meet the added expense for printing without cutting down the number of pages in the MONTHLY and this at a time when the amount of high-grade material on hand for publication is much greater than ever before, and when plans were already developing to increase the number of issues to eleven and eventually to twelve by adding one or two numbers devoted mainly to expository and historical articles of an elementary character.

But even all this is by no means the most serious phase of the situation. It is clear that the time has come when we should no longer allow one or two members of the Association, merely as a labor of love, to perform the very arduous services which devolve upon them. This was doubtless unavoidable in the early years of our development before the full significance of the movement which the MONTHLY inaugurated and the Association perpetuated, was widely comprehended. But the sacrifices and whole-hearted devotion given to a cause by its pioneers should not be presumed upon as indicating the normal method of procedure when the cause has become well established and its supporters are large in number.

To be more explicit, the responsibilities carried by our Secretary-Treasurer and by our Editor-in-Chief are burdensome beyond the belief of anyone who has not had actual experience in such matters, and we have no justification as a body in allowing such time and strength-consuming service to be given without compensation. We would all declare with enthusiasm that such service is invaluable and that its influence is of far-reaching importance, but we should also realize that the time given to it—amazing in amount, if we only knew all the facts—is taken at the expense of necessary recreation or of professional growth and advancement or of both. All of us expect to render a reasonable amount of altruistic service, but few of us are in a position to afford to devote a very large fraction of our time to such service for more than a brief period during some stress of circumstances, such, for instance, as happened during the war. In a word, I am advocating that the secretaryship and the chief editorship in such organizations as the Association and the Society should be offices with at least a moderate stipend attached. This, of course, is impossible at present in the Association, and always will be impossible so long as the dues are kept within the present range of reasonable reach of the average member.

One thing, however, the Council has decided must be done, namely, adequate paid assistance must be provided for these offices and the funds therefor must be found. Some paid assistance has, of course, been provided, but quite inadequate, either for the Secretary's office or for the editorial office. The Council

has voted to pay $100 per month, beginning at once, for expert editorial and clerical assistance in Professor Archibald's office. This will cancel all our reserve before 1922, unless we secure other income than that derived from the dues. This I believe can be done. A temporary subsidy fund can be raised by the voluntary contributions of members or of other friends of mathematics. When it becomes known that a definite service may be rendered to the cause of mathematics by such contributions, interested persons will come forward and offer assistance. Such contributions will tide over a present difficult situation, but what is needed is a permanent endowment fund. This also can be secured, but it will take more time. The reason that such endowments have not been made before is that the need and the opportunity have not been known. Steps have already been taken to incorporate the Association so that we may be legally authorized to handle such trust funds.[1] An endowment can be built up from many sources as the years advance. The Association will thus become able to pay for important service rendered without advancing dues beyond the reach of the average college teacher, and it will be able to magnify its work and influence in a multitude of ways, some of which have been described above, but more of which will develop as those in hand are worked out. Opportunities will be found for service not only in the current and routine affairs of the Association, but also in the greater undertakings such as the proposed Dictionary of Mathematical Terms, which is already definitely projected and awaiting funds, or a biographical dictionary or a history of mathematics, which may later become definite projects, any one of which would constitute an enduring monument to the person who should provide the funds to make such a project attainable. It seems not too much to hope that within a reasonable time the Association, perhaps in combination with the Society, should have a central office where its business and editorial affairs of whatever magnitude may be conducted efficiently and with just compensation to those who render such important service.

I have thus endeavored to describe some of the mathematical activities in America during the past twenty years and to show the relation of the Association to these activities during its brief history, first in regard to research scholarship in its broader sense and secondly, in regard to the teaching of mathematics in all its phases. I have tried to show that the Association has put into effective operation a really national investigation of the mathematical situation in the schools, conducted by experts in both teaching and scholarship whose findings and recommendations must inspire confidence and promote reforms wherever they may be needed. And, finally, I have tried to emphasize the pressing need of financial support for such an organization as the Association through subsidies and endowments which shall make possible the conduct of its great altruistic service without, on the one hand, unjustly overburdening the volunteer workers, and without, on the other hand, raising the annual dues beyond the reasonable reach of large numbers to whom the service of the Association is especially helpful. It is the privilege and duty of every member of the Association to make known this need and opportunity as widely as circumstances permit.

[1] The legal steps have since been completed.

THE GRAPHICAL SOLUTION OF SPHERICAL TRIANGLES.

BY HARRY C. BRADLEY, Massachusetts Institute of Technology.

Corresponding to every spherical triangle, there exists at the center of the sphere a trihedral angle, whose face angles are equal to the sides of the spherical triangle, and whose dihedral angles are equal to the angles of the triangle. Graphical solutions of the trihedral are readily obtained by descriptive geometry.

Gaspard Monge, the French genius who may be called the father of descriptive geometry, does not appear to have given us any solutions for the trihedral. At any rate, an edition of his descriptive geometry, dated 1820, the earliest edition to which I have had access, omits the subject of trihedrals entirely. The excellent work published by Prof. Albert E. Church of the U. S. Military Academy, West Point, in 1864, contains graphical solutions of the trihedral corresponding to all six cases which arise in the solution of spherical triangles, and is the earliest dated work which I have as yet discovered. A number of modern texts give these solutions. A list (far from complete) appears at the end of this article.[1]

[1] An early reference to the graphical solution of spherical triangles, similar to that of Professor Bradley's paper, is Chapter 17 ("Résolution des Triangles sphériques par la Regle et le Compas") in A. Cagnoli's *Traité de Trigonométrie rectiligne et sphérique* . . . traduit de l'Italien par M. Chompré. Paris, 1786; chapter 19 of the second edition, Paris 1808. (*Trigonometria piana e sferica*. Edizione seconda notabilmente ampliate. Bologna, 1804: "Risoluzioni de'triangoli sferici con la riga e col compasso," pp. 346–349). For similar and allied discussions the following sources may be consulted.

C. Gudermann, *Lehrbuch der niederen Sphärik*, Münster, 1836.

G. K. L. von Littrow, "Ueber Herrn M. Eble's graphische Methoden der Auflösung sphärischen Dreiecke mit besonderer Rücksicht auf sein neuestes 'Stundenzeiger' oder 'Horoskop' genanntes Instrument," *Sitzungsberichte der math.-naturwiss. Klasse d. k. Akademie der Wissen.*, Vienna, vol. 42, 1860, pp. 203–212.

F. C. Penrose, "Description of an improved diagram for the graphical solution of spherical triangles, applicable to the questions arising out of the spheroidal figure of the earth," *Monthly Notices of the Royal Astronomical Society*, vol. 37, May, 1877, pp. 403–409.

L. Janse, "Over het graphisch oplossen van bolvornigé driehoeken en van daarop gegronde zeevaart en sterrekundige vraagstukken," *Nieuw Archief voor Wiskunde*, vol. 11, 1884, pp. 1–27; vol. 12, 1886, pp. 113–148.

C. H. Smith, "A graphic method of solving spherical triangles," *Amer. Jl. Math.*, vol. 6, 1884, pp. 175–176.

C. Wiener, *Lehrbuch der darstellenden Geometrie*, vol. 1, Leipzig, 1884, pp. 104–113.

P. Braun, "Das Trigonometer von C. Braun," *Math. naturwissensch. Ber. aus Ungarn*, reprinted in W. Dyck, *Katalog mathematischer und mathematischphysikalischer Modelle, Apparate und Instrumente*, München, 1892, pp. 160–161.

M. D'Ocagne, (*a*) *Journal de l'Ecole Polytechnique*, second series, vol. 4, 1898, p. 224; (*b*) *Traité de Nomographie*, Paris, 1899, pp. 326–330.

S. Haller, "Beitrag zur Geschichte der konstruktiven Auflösung sphärischer Dreiecke durch stereographische Projektion," *Biblioteca Mathematica*, n.s., vol. 13, 1899, pp. 71–80.

F. N. Willson, *Theoretical and practical Graphics*, New York, 1902, pp. 206–210.

M. D'Ocagne, "Sur la résolution nomographique des triangles sphériques," (*a*) *Bull. Soc. Math. de France*, vol. 32, 1904, pp. 196–203; (*b*) *Comptes rendus de l'Académie des Sciences*, vol. 138, 1904, pp. 70–72.

G. Pesci, "Resolução nomographica do triangulo de Posição" [translated from the Italian into Portuguese by Radler de Aquino], *Revista Maritima Brazileira*, Nov.–Dec., 1907; Feb., 1908.

These graphical solutions, however, ordinarily appear merely as exercises in descriptive geometry. Without some modification or adaptation, they are not suited for general use as an aid in checking numerical computations of spherical triangles. Usually all parts of the triangle are taken as less than 90°. When several of the parts lie between 90° and 180°, the resulting construction often becomes very difficult, even with an expert knowledge of descriptive geometry, to execute and to interpret correctly. Yet, as a check to numerical computations, the graphical solutions are not without value. Especially is this true in those cases where ambiguity exists in the numerical solution. The graphical construction shows clearly one solution, two solutions, or none; and in the case of two solutions, the correspondence of the parts.

To test the accuracy which may be expected from a graphical solution, I drew a hundred or so figures of moderate size, say six or seven inches across. The angles were measured with a semi-circular protractor, five inches in diameter, and were laid out only to the nearest whole degree. Disregarding some extraordinary agreements, probably more or less accidental, an average accuracy of 1° or 2° was readily obtained. This is quite sufficient to detect any gross error of calculation.

Now, as an aid to checking numerical calculations, the fewer graphical solutions which can be made to serve, the better. Fortunately, direct graphical solutions of all possible cases of spherical triangles, with all possible combinations of acute and obtuse angles for given parts, are unnecessary, provided we are willing to combine a little trigonometry with our descriptive geometry. The trigonometry required is of the simplest sort, namely:

1. The principle of polar triangles, by which sides are replaced by the supplements of angles, and angles by the supplements of sides. For instance, the graphical solutions for three given sides are simple and direct, while those for three given angles are not. Hence, by applying the principle of polar triangles,

R. de Aquino, "Nomograms for deducing altitude and azimuth and for star identification and finding course and distance in great circle sailing," *U. S. Naval Institute Proceedings*, vol. 34, 1908, pp. 633–646. See also W. C. P. Muir, *Treatise on Navigation*, fourth edition, Annapolis, Md., 1918, Appendix D, pp. 773–777: "Solution of the astronomical triangle by nomography."

G. Pesci, "Cenni sulla risoluzione del triangolo di posizione senza calcoli trigonometrici," *Rivista Marittima*, Roma, vol. 42, Sept., 1909, pp. 317–328 + 1 table. [Discusses the "compasso trigonometrico" of G. F. Richer described by F. Callet in the supplement to Bézout's Spherical Trigonometry and Navigation, Paris, 1798].

G. Pesci, "Sulla risoluzioni dei triangoli sferici senza calcoli trigonometrici" *Supplemento al Periodico de Matematica*, Livorno, vol. 13, 1910, pp. 65–73.

C. Schoy, *Beiträge zur konstruktiven Lösung sphärischastronomischer Aufgaben*. Leipzig, 1910. 7 + 40 pp. + 8 plates.

G. Loria, *Vorlesungen über darstellende Geometrie*, translated by F. Schütte, part 2, Leipzig, 1913, pp. 3–15.

M. D'Ocagne, *Cours de Géométrie*, tome 2, Paris, 1918, pp. 312–314.

H. G. G., "A new graphic method in nautical astronomy," *Nature*, vol. 102, Oct. 24, 1918, pp. 155–6.

A. Hutchinson and H. B. Goodwin, "Graphic methods in astronomy," *Nature*, vol. 103, March 13, 20, 1919, pp. 25, 44. Graphic methods here employed are "recommended to crystallographers."

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both cases can be solved graphically by the construction for three given sides.

2. The principle of co-lunar triangles. If the given parts are of such magnitudes that the direct graphical solution is difficult of execution and interpretation, one, at least, of the three possible co-lunar triangles will give a simpler solution.

The problem which I set myself, then, was to find: 1. Which of the graphical solutions were simplest to construct and easiest to interpret; 2. The minimum number of such solutions necessary, after making any trigonometrical transformations in the data which seemed desirable. I submit the following constructions as my result. There is nothing especially new in them. Under the circumstances, there could not be. But one or two of them differ more or less in detail from anything I have ever happened to see published.

These constructions were all made by descriptive geometry. Those familiar with that subject will readily recognize the horizontal and vertical coördinate planes, the ground line, traces of oblique planes, and revolution about those traces. The constructions, however, may readily be made without knowledge of descriptive geometry, and to render them more generally useful will be described merely as constructions.[1] In each example, the angles of the given triangle are A, B, C, the opposite sides a, b, c. To make the examples more specific, actual numerical values (in degrees) for the parts of the triangle will be used throughout.

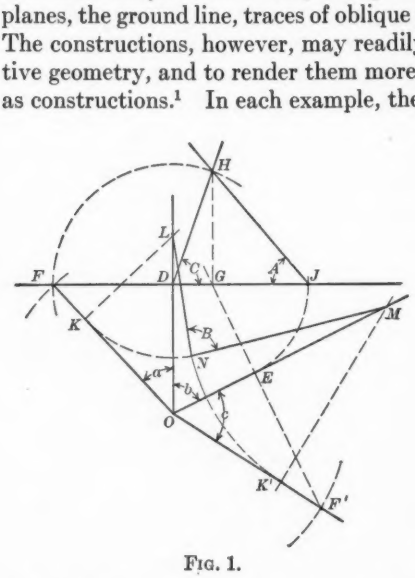

FIG. 1.

Case I. Given the three sides.

Example 1. All given sides acute. Given $a = 43°$, $b = 64°$, $c = 58°$.

This is solved in Fig. 1. Select any point O, and draw a vertical line OD. To the left of this line, lay off the smallest of the three given sides, in this case $a = 43°$. To the right of OD, lay off the largest side, $b = 64°$. Beyond this, lay off the remaining side, $c = 58°$. Draw a horizontal line intersecting the sides of a at F and D. Make $OF' = OF$. From F', draw $F'E$ perpendicular to OE, intersecting FD at G. At G, draw a vertical line. With D as center, radius DF, draw an arc intersecting this vertical at H. Draw HD. Then HDG is the angle $C = 70°$. With G as center, radius GE, draw an arc to intersect FD at J. Draw HJ. Then HJG is the angle $A = 49°$.

To find the remaining angle, opposite the middle side b, take any point K on OF. Draw from K a perpendicular to OF to meet OD at L. Make $OK' = OK$. From K' draw $K'M$ perpendicular to OF' to meet OE at M. With L as center,

[1] We have in this article a good illustration of the employment of the methods of descriptive geometry in pure mathematics as recommended very strongly by Professor Gino Loria in letters to Professor Roever published last year in this MONTHLY (*1919*, 45–53).—EDITOR.

radius LK, draw an arc. With M as center, radius MK', draw an arc, meeting the first arc at N. Then angle LNM is the angle $B = 85°$.

Answer. $A = 49°$, $B = 85°$, $C = 70°$.

Example 2. Two given sides acute, one obtuse. Given $a = 60°$, $b = 126°$, $c = 76°$.

The construction is shown in Fig. 2. It differs from Fig. 1 only in the size of the given parts. The figure is lettered the same as Fig. 1, and the same directions will apply.

Answer. $A = 35°$, $B = 148°$, $C = 40°$.

Example 3. Two given sides obtuse, one acute. Given $a = 60°$, $b = 100°$, $c = 140°$.

Form the co-lunar triangle $a = 60°$, $b' = 180° - b = 80°$, $c' = 180° - c = 40°$. Solve this by the construction of Fig. 1. This gives $A = 55°$, $B' = 112°$, $C' = 37°$. Whence, taking the co-lunar triangle of this result, we have the

Answer. $A = 55°$, $B = 68°$, $C = 143°$.

Example 4. All given sides obtuse. Given $a = 100°$, $b = 110°$, $c = 135°$.

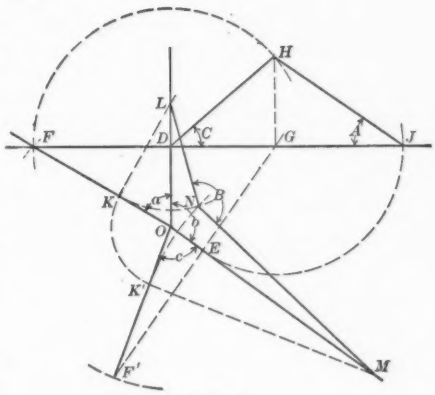

FIG. 2.

Form the co-lunar triangle $a = 100°$, $b' = 70°$, $c' = 45°$. Solve this by the construction of Fig. 2, giving $A = 129°$, $B' = 48°$, $C' = 34°$. Whence,

Answer. $A = 129°$, $B = 132°$, $C = 146°$.

Example 5. An impossible case. Given $a = 30°$, $b = 80°$, $c = 40°$.

If we overlook the fact that $a + c < b$, the construction will bring us to our senses. Proceeding as in Fig. 1, we find point G, and draw a vertical line at this point. But with the parts as given, DG will be greater than DF. Hence, when we take D as center, DF as radius, and draw an arc, the arc will not intersect the vertical line at G. Therefore there is no solution.

Case II. Given the three angles.

As already stated, no direct constructions for this case are needed. Take the polar triangle corresponding to the given triangle, and solve by Case I.

Case III. Given two sides and the included angle.

Example 6. All given parts acute. Given $a = 43°$, $b = 64°$, $C = 70°$.

This is the same triangle solved from different given parts in Fig. 1, and the completed solution will be the same figure. To construct the figure from the above given parts, draw a vertical line OD. To the left of OD lay off the smaller of the given sides, $a = 43°$. To the right, lay off $b = 64°$. Draw the horizontal line FD. At D lay off the given angle C as shown. With D as center, radius

DF, draw an arc to intersect DH at H. Draw a vertical line HG, to meet FD produced at G. Draw GE perpendicular to OM. With O as center, radius OF, draw an arc to intersect GE produced at F'. Draw OF'. Then EOF' is the third side, $c = 58°$. The three sides now being known, the remaining angles, $A = 49°$ and $B = 85°$, are found as previously explained.

Answer. $c = 58°$, $A = 49°$, $B = 85°$.

Example 7. The two given sides acute, the given angle obtuse. Given $a = 61°$, $b = 53°$, $C = 135°$.

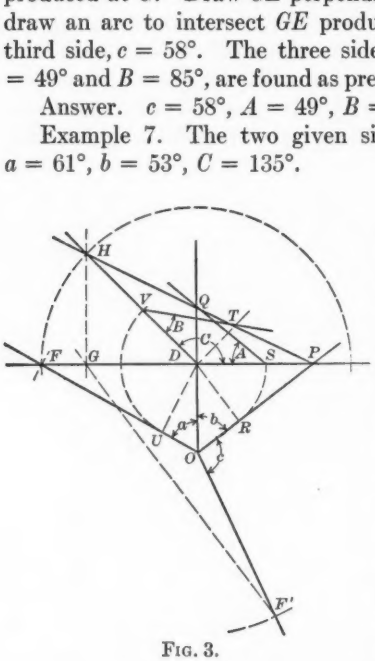

FIG. 3.

This example is solved in Fig. 3. Draw the vertical line OD. To the left of OD lay off the *larger* of the given sides, $a = 61°$. To the right of OD lay off the smaller given side, $b = 53°$. Draw a horizontal line FD, intersecting the sides of the angle a at F and D. Produce FD to intersect the remaining side of angle b at P; this can always be done, since the smaller side has been placed at the right of OD. At D, lay off the given angle $C = 135°$, as shown. With center D, radius DF, draw an arc intersecting DH at H. From H draw a vertical line to intersect FD at G. From G draw a line perpendicular to OP. With O as center, radius OF, draw an arc to intersect this line at F'. Then POF' is the third side, $c = 102°$.

Connect H with P, intersecting OD produced at Q. Draw DR perpendicular to OP. On DP, make distance $DS = DR$. Then DSQ is the angle opposite the side DOF, $A = 39°$.

From D, draw DT perpendicular to DH, intersecting HP at T. Also draw DU perpendicular to OF. Lay off on DH the distance $DV = DU$. Draw VT. Then DVT is the angle opposite the middle side, $B = 35°$.

Answer. $c = 102°$, $A = 39°$, $B = 35°$.

No other figures are needed for this Case. Any other conditions can be reduced to one of the two preceding solutions by the use of a co-lunar triangle. (See Examples 3 and 4.)

Case IV. Given two angles and the included side.

Take the polar triangle corresponding to the given triangle, and solve by Case III.

There are no impossible cases under Cases III and IV, and the construction can always be made.

Case V. Given two sides and the angle opposite to one of them.

Example 8. All three given parts acute. Given $a = 45°$, $b = 58°$, $A = 39°$.

The construction is shown in Fig. 4. Draw the vertical line OD. To the

left of OD lay off the side whose opposite angle is given, $a = 45°$. To the right of OD, lay off the other given side, $b = 58°$. Draw a horizontal line to give intersections F, D, and P. Draw DR perpendicular to OP. On DP, make $DS = DR$. At S, lay off the given angle A as shown, obtaining the intersection Q on OD produced. Draw PQ. With D as center, radius DF, draw an arc to intersect PQ. Since the side a is less than the side b, $DF < DP$, and the arc will intersect PQ in two points to the left of P, namely, H and H', each of which will give a solution to the problem.

For the first answer, from H draw a vertical line, intersecting DP at G. From G draw GE perpendicular to OP. With center O, radius OF, intersect this line at F'. Then EOF' is the third side, $c_1 = 18°$. Draw DH. Then GDH is the angle opposite this side, $C_1 = 16°$. The angle opposite the middle side, b, is found at N by the construction explained in Fig. 1; $B_1 = 131°$.

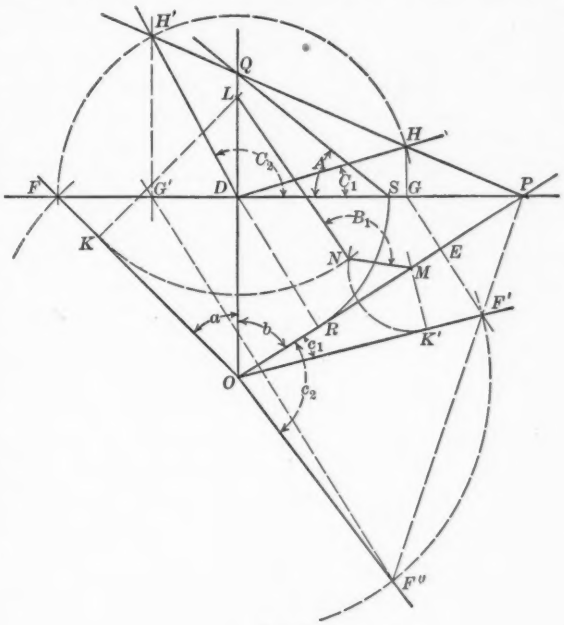

FIG. 4.

First Answer. $c_1 = 18°, C_1 = 16°, B_1 = 131°$.

Proceeding similarly with the point H', we find EOF'' is $c_2 = 84°$, and GDH' is $C_2 = 118°$. No construction is needed for the remaining angle, B_2, since this angle is known by trigonometry to be the supplement of B_1. Hence $B_2 = 49°$.

Second Answer. $c_2 = 84°, C_2 = 118°, B_2 = 49°$.

Note. Should the intersection at F' prove rather flat, the point F' may be located more accurately from the fact that it lies on the line PF'', as shown in the figure.

Example 9. Given $a = 57°$, $b = 63°$, $A = 70°$.

If we proceed as explained for Fig. 4, the circle with D as center, radius DF, will be found to be tangent to PQ. There is then but one point H, and one solution. Going on with the construction from the point H, the third side is EOF', $c = 34°$. The angle opposite this side is GDH, $C = 38°$. The angle opposite the middle side needs no construction; by trigonometry, $B = 90°$.

Answer. $c = 34°, B = 90°, C = 38°$.

Example 10. Given $a = 38°$, $b = 59°$, $A = 53°$.

If we proceed as before, we shall find in this example that the radius DF is so short that the arc from F will not intersect PQ. There is consequently no solution.

Example 11. Given $a = 61°$, $b = 53°$, $A = 39°$.

This is the same triangle that was solved from different given parts in Fig. 3, and the completed construction will be the same figure.

The construction is started as explained for Fig. 4, Ex. 8, and proceeds as there explained until we draw the arc with D as center, radius DF. In this example, the side a, opposite the given angle A, is greater than the other given side b. Hence $DF < DP$, and the arc drawn from F will intersect the line PQ at but one point to the left of P. This intersection, H, gives the only solution to this example. Draw DH. Then PDH is the angle included between the given sides a and b, $C = 135°$.

Two sides and the included angle being known, the construction can now be completed as explained in case III. See Example 7.

Answer. $c = 102°$, $B = 35°$, $C = 135°$.

Example 12. Given $a = 63°$, $b = 58°$, $A = 75°$.

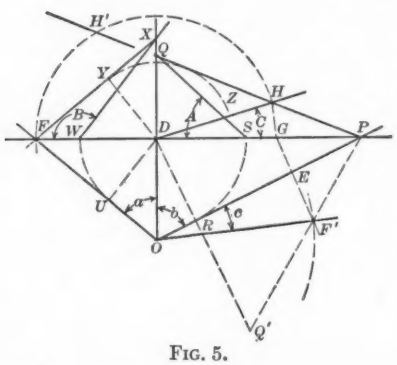

FIG. 5.

This is apparently the same case as that of the preceding example, since the side opposite the given angle is larger than the other. However, on attempting the construction, the given parts will be found to be of such sizes that the intersection H lies between P and Q. The angle included between the given sides is therefore acute, and the construction is best completed by the method used in Fig. 4 for the point H. See Example 8.

Answer. $c = 60°$, $B = 67°$, $C = 70°$.

Example 13. The two given sides acute, the given angle obtuse. Given $a = 51°$, $b = 64°$, $B = 129°$.

The construction is shown in Fig. 5. Draw the vertical line OD. At the *right* of this line, lay off the side whose opposite angle is given, $b = 64°$. Lay off the other side, $a = 51°$, to the left of OD. Draw a horizontal line to give the intersections F, D, and P. From D draw DU perpendicular to OF. Make the distance $DW = DU$. At W lay off the given angle, $B = 129°$, as shown, giving the intersection X on OD produced. Draw FX. From D draw DY perpendicular to FX. With D as center, radius DY, draw the arc YZ. From P draw the line PQ tangent to this arc, intersecting OD produced at Q.

With D as center, radius DF, draw an arc to intersect PQ. The intersection H', nearer F, does not give a solution to the problem. A solution exists only if a second intersection, H, can be found on PQ to the left of P. It is evident that this

can only occur when, as in the figure, $b > a$. If $b < a$, P will be inside the circle with radius DF, there will be no second intersection H, and no solution.

From H draw the vertical line HG, intersecting DP at G. From G draw GE perpendicular to OP. With O as center, radius OF, intersect this perpendicular at F'. Then EOF' is the third side, $c = 19°$. Draw HD. Then HDG is the angle opposite this side, $C = 16°$.

From D draw DR perpendicular to OP. Make the distance $DS = DR$. Draw SQ. Then DSQ is the remaining angle, $A = 42°$.

In case the intersection at F' should be too flat to be accurate, the point F' may be located as follows. Produce DR, which is perpendicular to OP. Make the distance $RQ' = SQ$. Draw $Q'P$; this line passes through F'.

Answer. $c = 19°$, $A = 42°$, $C = 16°$.

Any other example under this case can be reduced to one of those already given by the use of a co-lunar triangle.

Case VI. Given two angles and the side opposite to one of them.

Take the polar triangle corresponding to the given triangle, and solve by Case V.

Summary of the Constructions. In all the foregoing constructions, two of the given parts are always two sides, and acute. The constructions may then be summarized according to the third given part, as follows.

1. The third side, acute. Each side less than the sum of the other two. Fig. 1. One solution.

2. The third side, obtuse, but less than the sum of the other two. Fig. 2. One solution.

3. The third side, one of the three sides equal to, or greater than, the sum of the other two. No solution.

4. Angle included between the given sides, acute. Fig. 1. One solution.

5. Angle included between the given sides, obtuse. Fig. 3. One solution.

6. Angle opposite the smaller given side, acute. Fig. 4. Two solutions, one solution, or no solution.

7. Angle opposite the larger given side, acute. Start Fig. 4 and find the angle which is included between the given sides. If this angle is acute, complete Fig. 4. But if the included angle is obtuse, use the construction of Fig. 3. One solution.

8. Angle opposite the larger given side, obtuse. Fig. 5. One solution.

9. Angle opposite the smaller given side, obtuse. No solution.

Any other example is reduced to one of the above by suitable trigonometric transformations.

References. In the following, the solutions are given as problems in descriptive geometry, and a knowledge of that subject is required to understand them.

A. E. Church, *Elements of Descriptive Geometry*, New York, 1864.

C. Margerie et E. Racine, *Traité de Géométrie Descriptive*, Paris, 1883.

C. A. Waldo, *Manual of Descriptive Geometry*, Boston, 1895.

F. Chomé, *Cours de Géométrie Descriptive de l'Ecole Militaire*, première partie, Paris, 1898.

W. S. Hall, *Descriptive Geometry*, New York, 1903.
S. E. Warren, *Elements of Descriptive Geometry*, New York, 1905.
J. B. Millar, *Elements of Descriptive Geometry*, New York, 1905.

QUESTIONS AND DISCUSSIONS.

EDITED BY W. A. HURWITZ, Cornell University, Ithaca, N. Y.

REPLIES.

34 [*1917*, 134, 341; *1920*, 114, 301, 405]. Given the mixed integral and functional equation

$$\int_{x=0}^{x=h} f(x)dx = \frac{h}{6}\left[f(0) + 4f\left(\frac{h}{2}\right) + f(h)\right],$$

to determine the function $f(x)$. This equation is of rather fundamental practical value as it has to do with the most general solid whose volume is given by the prismatoid formula.

I. REMARKS BY A. A. BENNETT, Baltimore, Md.

For the functional equation of Simpson's Rule, the origin is a point specially characterized. If smoothness at the origin be not required, solutions which are continuous and contain an infinite number of parameters may be obtained which present irregularities at this point. In particular the following one-parameter family of analytic functions with an essential singularity at the origin satisfy the equation:

$$x^a \sin(b \log x - c),$$

where a and b are constants satisfying certain transcendental equations and where c is an arbitrary parameter. This solution may furthermore be added to any other solution to yield a solution.

For the equation

$$\int_0^h f(x)dx = \frac{h}{6}\left[f(0) + 4f\left(\frac{h}{2}\right) + f(h)\right],$$

if f is differentiable,[1] and $f(0) = 0$, becomes, after differentiation

$$6f(x) = [4f(x/2) + f(x)] + x[2f'(x/2) + f'(x)].$$

Testing $f(x) \equiv x^a \sin(b \log x - c)$ we have

$$f'(x) = x^{a-1}[a \sin(b \log x - c) + b \cos(b \log x - c)].$$

Substituting throughout this gives

$$\begin{aligned} 6x^a \sin(b \log x - c) = (x^a/2^{a-2}) \sin(b \log x/2 - c) + x^a \sin(b \log x - c) \\ + (x^a/2^{a-2})[a \sin(b \log x/2 - c) + b \cos(b \log x/2 - c)] \\ + x^a[a \sin(b \log x - c) + b \cos(b \log x - c)]. \end{aligned}$$

Collecting terms, and canceling x^a, this yields

$$\begin{aligned} (5 - a) \sin(b \log x - c) - b \cos(b \log x - c) \\ = (1/2^{a-2})[(a + 1) \sin(b \log x/2 - c) + b \cos(b \log x/2 - c)]. \end{aligned}$$

Writing $b \log x/2 - c$ as $(b \log x - c) - b \log 2$ and expanding the terms in which this occurs, then equating coefficients of $\sin(b \log x - c)$ and of $\cos(b \log x - c)$, one obtains

$$\begin{cases} (5 - a) \cdot 2^{a-2} = (a + 1) \cos(b \log 2) + b \sin(b \log 2) \\ b \cdot 2^{a-2} = -b \cos(b \log 2) + (a + 1) \sin(b \log 2). \end{cases}$$

These equations are satisfied by an infinite set of values of a, b.[2]

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[1] The assumption of differentiability is not essential. Direct substitution of the function $x^a \sin(b \log x - c)$ in the given equation leads to a pair of conditions on a, b equivalent to those derived in the text.

[2] See the remarks by the Editor, immediately following the present article.

In order to exhibit the special rôle of the point $x = 0$, we shall obtain from the given equation another not involving this point; and then examine the nature of solutions of the new equation and under what conditions the old equation can be derived from the new. We assume first that $f(x)$ is continuous, and start with the given equation.

$$\int_0^x f(t)dt = \frac{x}{6}[f(0) + 4f(x/2) + f(x)]. \tag{1}$$

This is a special case of

$$\int_0^x f(t)dt = \frac{x}{6}[C + 4f(x/2) + f(x)]. \tag{2}$$

In (2), give x the constant value $a \neq 0$; thus

$$\int_0^a f(t)dt = \frac{a}{6}[C + 4f(a/2) + f(a)]; \tag{3}$$

and solve for C, thus

$$C = \frac{6}{a}\int_0^a f(t)dt - 4f(a/2) - f(a). \tag{4}$$

Substitute in (2),

$$\int_0^x f(t)dt = \frac{x}{6}\left[\frac{6}{a}\int_0^a f(t)dt - 4f(a/2) - f(a) + 4f(x/2) + f(x)\right]. \tag{5}$$

In (5), replace x by the constant $2a$,

$$\int_0^{2a} f(t)dt = \frac{a}{3}\left[\frac{6}{a}\int_0^a f(t)dt - 4f(a/2) + 3f(a) + f(2a)\right]. \tag{6}$$

Subtract (5),

$$\int_x^{2a} f(t)dt = \left(2 - \frac{x}{a}\right)\int_0^a f(t)dt - \tfrac{2}{3}(2a - x)f(a/2) + \tfrac{1}{6}(6a + x)f(a) + \tfrac{1}{3}af(2a) - \tfrac{2}{3}xf(x/2) - \tfrac{1}{6}xf(x). \tag{7}$$

From (6), subtract $\int_0^a f(t)dt$, giving

$$\int_a^{2a} f(t)dt = \int_0^a f(t)dt - \tfrac{4}{3}af(a/2) + af(a) + \tfrac{1}{3}af(2a) \tag{8}$$

or

$$\int_0^a f(t)dt = \int_a^{2a} f(t)dt + \tfrac{4}{3}af(a/2) - af(a) - \tfrac{1}{3}af(2a). \tag{9}$$

Substitute in (7), giving the equation

$$\int_x^{2a} f(t)dt = \left(2 - \frac{x}{a}\right)\int_a^{2a} f(t)dt + \tfrac{2}{3}(2a - x)f(a/2) - \tfrac{1}{6}(6a - 7x)f(a) - \tfrac{1}{3}(a - x)f(2a) - \tfrac{2}{3}xf(x/2) - \tfrac{1}{6}xf(x). \tag{10}$$

Solving for $4xf(x/2)$, we have

$$4xf(x/2) = -xf(x) - 6\int_x^{2a} f(t)dt + 6\left(2 - \frac{x}{a}\right)\int_a^{2a} f(t)dt + (8a - 4x)f(a/2) - (6a - 7x)f(a) - (2a - 2x)f(2a). \tag{1'}$$

For the new equation (1') we have the following

THEOREM: *Given any positive number, a, and any real one-valued function continuous in the interval $I : a \leqq x \leqq 2a$, the end points being included, and further given an arbitrary value, A,—there exists a unique solution of* (1) *coinciding with the given function in I, taking on the value A at $a/2$ and continuous for all real positive values of x.*

It is at once obvious that the value of $f(1')$ in $I : a \leqq x \leqq 2a$ and its value at $a/2$ determine from (1') its value in $a/2 \leqq x \leqq a$; by substitution of $x = 2a$ in (1') it appears that $f(x)$ is continuous in both directions at $x = a$. By the methods of integral equations the value of $f(x)$ is determined by (1') in $2a \leqq x \leqq 4a$. By repetition of this process the definition of the function can be extended repeatedly to intervals on the left, each half as long as the preceding, and to intervals on the right, each twice as long as the preceding. It need not be true, however, that the values so obtained on the left approach any limit at all as x approaches zero.

In order to return from (1′) to (1) we make the following assumptions:
(I) $f(x)$ is continuous, $x > 0$, and satisfies (1′) for some value of the constant a.
(II) $f(x)$ is bounded in a neighborhood of $x = 0$.
(III) If $\int_0^x f(t)dt$ can be written in the form $\frac{x}{6}\left[C + 4f\left(\frac{x}{2}\right) + f(x) \right]$, then $f(0)$ exists and $C = f(0)$.

The condition (II) may be replaced by the two weaker conditions:
(IIa) $\lim_{x \to 0} xf(x) = 0$,
(IIb) $\int_0^x f(t)dt$ converges for $x > 0$.

Condition (III) may be replaced by
(IIIa) $f(0)$ is defined and $f(x)$ is continuous at $x = 0$. If (IIIa) is assumed, (II) is superfluous.

Starting with

$$4xf(x/2) = -xf(x) - 6\int_x^{2a} f(t)dt + 6\left(2 - \frac{x}{a}\right)\int_a^{2a} f(t)dt + (8a - 4x)f(a/2) - (6a - 7x)f(a) - (2a - 2x)f(2a) \quad (1')$$

we have, upon letting x approach 0,—thus utilizing condition (II), or (IIa) and (II$_b$)—the following.

$$0 = -6\int_0^{2a} f(t)dt + 12\int_a^{2a} f(t)dt + 8af(a/2) - 6af(a) - 2af(2a),$$

whence we reobtain,

$$\int_0^a f(t)dt = \int_a^{2a} f(t)dt + \tfrac{4}{3}af(a/2) - af(a) - \tfrac{1}{3}af(2a). \quad (9)$$

Eliminating $\int_a^{2a} f(t)dt$ between (9) and (1′) we have again

$$\int_x^{2a} f(t)dt = \left(2 - \frac{x}{a}\right)\int_0^a f(t)dt - \tfrac{2}{3}(2a - x)f(a/2) + \tfrac{1}{6}(6a + x)f(a) + \tfrac{1}{3}af(2a) - \tfrac{2}{3}xf(x/2) - \tfrac{1}{6}xf(x). \quad (7)$$

In (7), letting x approach 0, we obtain

$$\int_0^{2a} f(t)dt = \frac{a}{3}\left[\frac{6}{a}\int_0^a f(t)dt - 4f(a/2) + 3f(a) + f(2a)\right]. \quad (6)$$

Subtracting (7) from (6), we have,

$$\int_0^x f(t)dt = \frac{x}{6}\left[\frac{6}{a}\int_0^a f(t)dt - 4f(a/2) - f(a) + 4f(x/2) + f(x)\right], \quad (5)$$

which is of the form

$$\int_0^x f(t)dt = \frac{x}{6}[C + 4f(x/2) + f(x)]. \quad (2)$$

By employing condition (III), we return to

$$\int_0^x f(t)dt = \frac{x}{6}[f(0) + 4f(x/2) + f(x)], \quad (1)$$

as desired.

Thus solutions of (1′) which satisfy (I), (II), (III) or (I), (IIa), (IIb), (III) or (I), (IIIa) will be solutions of (1). Solutions of (1′) include, however, functions not satisfying these further conditions bearing on the point $x = 0$ and therefore not serving as solutions of the original equation.

II. Remarks by the Editor.

It will be of value to indicate the relationship of the new solutions found by Professor Bennett to the work which has previously appeared in connection with this question. A convenient method of approach is found in the fact, used in earlier replies, that $f(x) = x^n$ is a solution of the

functional equation, in case n is a positive integer, if and only if

$$\frac{1}{n+1} - \frac{1}{6 \cdot 2^{n-2}} - \frac{1}{6} = 0. \tag{1}$$

We first extend our consideration to values of n, integral or fractional, real or complex. It is evident that for any n whose real part is positive, (1) still gives the condition that x^n be a solution. This may be written

$$\frac{n-5}{n+1} + \frac{1}{2^{n-2}} = 0;$$

or, letting

$$n - 2 = m,$$

$$\frac{m-3}{m+3} + 2^{-m} = 0. \tag{2}$$

In the equivalent statement

$$(m-3) + (m+3) \cdot 2^{-m} = 0$$

put

$$m = \alpha + i\beta,$$

and separate the real and pure imaginary parts of the resulting equation. Writing for brevity

$$\log 2 = l,$$

we have

$$(\alpha - 3) + 2^{-\alpha}[(\alpha + 3)\cos \beta l + \beta \sin \beta l] = 0, \tag{3}$$

$$\beta + 2^{-\alpha}[\beta \cos \beta l - (\alpha + 3)\sin \beta l] = 0. \tag{4}$$

Multiply (3) by $\sin \beta l$, (4) by $\cos \beta l$, and add:

$$(\alpha - 3)\sin \beta l + \beta \cos \beta l + 2^{-\alpha} \cdot \beta = 0. \tag{5}$$

Multiply (4) by 2^α, and subtract (5) from it:

$$\beta[2^\alpha - 2^{-\alpha}] - 2\alpha \sin \beta l = 0.$$

The latter equation may also be written

$$\beta \sinh \alpha l = \alpha \sin \beta l. \tag{6}$$

This equation can have no solutions for which both α and β differ from zero. In fact, if $\alpha \neq 0$, $\beta \neq 0$, (6) may be written

$$\frac{\sinh \alpha l}{\alpha l} = \frac{\sin \beta l}{\beta l};$$

but $(\sinh u)/u > 1$ and $(\sin u)/u < 1$, for all real $u \neq 0$, so that the equation cannot hold.

In case α or β vanishes, (6) holds. It remains to be seen when (3), (4) are satisfied. First let $\beta = 0$; then (4) is satisfied, and (3) reduces to (2), with α replacing m. This equation is readily seen to have the solutions $\alpha = -1, 0, 1$, corresponding to $n = 1, 2, 3$. It will be shown that there are no others. Writing the equation in the form

$$\frac{2^{\alpha/2}}{2^{-\alpha/2}} = \frac{3+\alpha}{3-\alpha},$$

we have

$$\frac{2^{\alpha/2} - 2^{-\alpha/2}}{2^{\alpha/2} + 2^{-\alpha/2}} = \frac{\alpha}{3},$$

or

$$\tanh \tfrac{1}{2}\alpha l = \tfrac{1}{3}\alpha.$$

This may be written

$$\tanh u - ku = 0, \tag{7}$$

where

$$u = \tfrac{1}{2}\alpha l, \qquad k = \frac{2}{3l}.$$

Now the function $\tanh u - ku$ vanishes for $u = 0$; if it vanished for two positive values of u, then by Rolle's theorem, its first derivative would vanish for at least two positive values, and its second derivative for at least one. But its second derivative is $-2 \tanh u \operatorname{sech}^2 u$ and never

vanishes for $u \neq 0$. Thus (7) has at most one positive solution, and similarly at most one negative solution.

Finally let $\alpha = 0$. Equations (4), (3) reduce to

$$3 \sin \beta l = \beta(1 + \cos \beta l),$$

$$\beta \sin \beta l = 3(1 - \cos \beta l);$$

and this pair of equations is equivalent to the single equation

$$3 \sin \tfrac{1}{2}\beta l = \beta \cos \tfrac{1}{2}\beta l.$$

Since it is clear that no value which makes $\cos \frac{1}{2}\beta l$ vanish can be a solution, the only admissible values of β are those for which

$$\tan \tfrac{1}{2}\beta l = \tfrac{1}{3}\beta, \tag{8}$$

which may be written

$$\tan u - ku = 0, \tag{9}$$

where

$$u = \tfrac{1}{2}\beta l, \qquad k = \frac{2}{3l}.$$

This is a type of equation which arises in many problems of analysis. Inspection of the graphs of $\tan u$ and ku leads at once to the well-known fact, which can readily be proved by rigorous analytic methods, that (8) has an infinite number of solutions, approximating more and more closely, as they increase numerically, to odd multiples of $\pi/2$.

We condense all these results into the statement: The function x^n, where n is a constant whose real part is positive, satisfies the proposed functional equation for the following values of n and no others:

(a) The real values $n = 1, 2, 3$.

(b) An infinite set of values of the form $n = 2 + \beta i$, where β is a solution[1] of (8).

Any of the second set of solutions leads to real solutions of the type considered by Professor Bennett. In fact, since

$$x^n = x^{a+2} = x^{a+2+\beta i} = x^{a+2} \cos(\beta \log x) + ix^{a+2} \sin(\beta \log x),$$

for any n yielding a complex solution we have the two real solutions $x^{a+2} \cos(\beta \log x)$ and $x^{a+2} \sin(\beta \log x)$, whence also the more general solution $x^{a+2} \sin(\beta \log x - c)$, which is a linear combination of them. Equations (3), (4) are equivalent to Professor Bennett's pair of equations for a, b with $a = \alpha + 2$, $b = \beta$. We have thus shown that Professor Bennett's formula gives a solution only (except for the trivial cases $b = 0$, $a = 1, 2, 3$) when $a = 2$, and that in this instance an infinite number of values of b will be possible. It is obvious that much more general solutions can be constructed, of the form $x^2F(x)$, where

$$F(x) = \Sigma[A_p \cos(\beta_p \log x) + B_p \sin(\beta_p \log x)],$$

proper precautions being taken to insure the validity of the series.

The importance of these new solutions lies in the following facts. The function $x^2 \sin(\beta \log x - c)$ possesses a continuous first derivative even at the origin. In previous editorial comment, it was stated that "it seems reasonable to adopt as a goal with reference to this question the proof that if the equation holds for some range of the variable h, the function $F(x)$ can only be a polynomial of degree $\leqq 3$, under restrictions as light as possible—*e.g.*, that $F(x)$ should be continuous and possess a stated number (as small as it can be made) of derivatives." The statement has now been proved, with the number of derivatives as great as *six*, by Professor Gillespie. Professor Bennett's example, together with the present comments, shows that the theorem is not true with the number of derivatives as small as *one*. Can the gap between these two results be bridged?

DISCUSSIONS.

In a previous number of the MONTHLY (*1920*, 53) Mr. W. F. Cheney gave an instance of a geometric proof of the law of tangents in trigonometry. In the first discussion below Professor Lovitt shows how a number of such proofs may be obtained. The two papers contain references to a number of other similar proofs.

[1] The smallest positive value of β is approximately 12.94.

In the usual form of the law of the mean for derivatives, $f(x+h) - f(x) = hf'(x+\theta h)$, the quantity θ depends on both x and h. It is natural to inquire into the character of the functional dependence of θ on x and h. A general investigation of this dependence has been made by Hedrick, *Annals of Mathematics*, series 2, p. 177. Professor Downing in the second discussion undertakes the more special task of studying the nature of θ for simple forms of the function $f(x)$. He takes up the cases of linear, quadratic, and cubic functions.

I. Geometrical Proofs of the Law of Tangents.

By W. V. Lovitt, Colorado College.

The geometrical proof depends in many cases upon finding pairs of lines the ratio of whose lengths is $(a-b)/(a+b)$, where a and b are sides of a given triangle. We proceed then to find such pairs of lines.

Make

$$AC = SC = CV = CR = b;$$

make

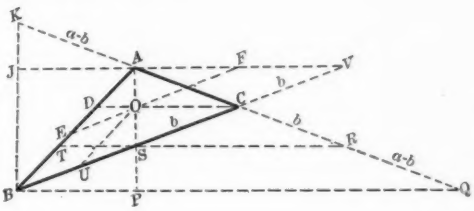
$$CK = CQ = CB = a.$$

Then

$$AK = BS = RQ = a - b,$$

and

$$BV = RK = a + b.$$



Draw CD bisecting the angle ACB. The lines VJ, TR, CD, and BQ are parallel. Draw AP and $BK \perp BQ$. Draw $EOF \parallel BC$ and $OU \parallel AB$.

Then

$$BU = \tfrac{1}{2}(a-b), \qquad UC = \tfrac{1}{2}(a+b),$$

and

$$\frac{a-b}{a+b} = \frac{BS}{BV} = \frac{EO}{EF} = \frac{BU}{UC} = \frac{DO}{OC} = \frac{TS}{AV} = \frac{TS}{SR} = \frac{BP}{PQ} = \frac{JA}{JV} = \frac{BS}{AQ} = \frac{AK}{BV}. \quad (1)$$

We also find

$$\tfrac{1}{2}(A+B) = \angle OSC = \angle OAC = \angle JBV = \angle JKA,$$

$$\tfrac{1}{2}(A-B) = \angle BAP = \angle JBA.$$

In many geometrical proofs of the law of tangents the only points of difference are in the precise figure and method used in establishing some one or more of the equalities (1). If equalities (1) were a well known part of the course in elementary geometry, then the several different proofs of the law of tangents would be simpler.

We give below some proofs which are believed to be new. With one excep-

tion they make use of the figure above. In each instance only such part of the figure need be actually drawn as is used in the proof.

(1) $$\frac{a-b}{c}=\frac{\sin\frac{1}{2}(A-B)}{\sin BSA}=\frac{\sin\frac{1}{2}(A-B)}{\sin\frac{1}{2}(A+B)},$$

$$\frac{a+b}{c}=\frac{\sin ABQ}{\sin AQB}=\frac{\cos BAP}{\sin C/2}=\frac{\cos\frac{1}{2}(A-B)}{\cos\frac{1}{2}(A+B)},$$

whence

$$\frac{a-b}{a+b}=\frac{\tan\frac{1}{2}(A-B)}{\tan\frac{1}{2}(A+B)}.$$

(2) $$\tan\tfrac{1}{2}(A-B)=\frac{BP}{AP}=\frac{(a-b)\sin BSP}{(a+b)\cos PAQ}=\frac{(a-b)\sin\frac{1}{2}(A+B)}{(a+b)\cos\frac{1}{2}(A+B)}.$$

$$=\frac{a-b}{a+b}\tan\tfrac{1}{2}(A+B).$$

(3) $$\angle SAC=\tfrac{1}{2}(A+B);\qquad \angle SAB=\tfrac{1}{2}(A-B);$$

$$EF\parallel BV;\qquad CD\perp AS;\qquad CD\parallel AV.$$

$$CS=CA=CV=b;\qquad BS=a-b;\qquad BV=a+b;$$

$$\tan\tfrac{1}{2}(A-B)=\frac{DO}{AO};\qquad \tan\tfrac{1}{2}(A+B)=\frac{CO}{AO};$$

$$\frac{\tan\frac{1}{2}(A-B)}{\tan\frac{1}{2}(A+B)}=\frac{DO}{CO}=\frac{DO}{AF}=\frac{EO}{EF}=\frac{BS}{BV}=\frac{a-b}{a+b}.$$

(4) $$\angle VBJ=\tfrac{1}{2}(A+B);\qquad \angle ABJ=\tfrac{1}{2}(A-B);$$

$$\angle VJB=\angle SAV=90^\circ;\qquad CA=CS=CV=b;$$

$$\tan\tfrac{1}{2}(A-B)=\frac{AJ}{BJ};\qquad \tan\tfrac{1}{2}(A+B)=\frac{VJ}{BJ};$$

$$\frac{\tan\frac{1}{2}(A-B)}{\tan\frac{1}{2}(A+B)}=\frac{AJ}{VJ}=\frac{BS}{BV}=\frac{a-b}{a+b},$$

or

$$\tan\tfrac{1}{2}(A-B)=\frac{AJ}{BJ}=\frac{BS\sin\frac{1}{2}(A+B)}{BV\cos\frac{1}{2}(A+B)}$$

$$=\frac{a-b}{a+b}\tan\tfrac{1}{2}(A+B).$$

(5) $$\angle ABJ=\tfrac{1}{2}(A-B);\qquad \angle JKA=\tfrac{1}{2}(A+B);\qquad \angle AJK=90^\circ;$$

$$CA=CV=b;\qquad AK=a-b;$$

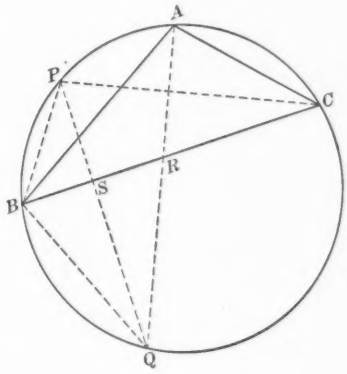
$$\tan \tfrac{1}{2}(A - B) = \frac{AJ}{BJ}; \qquad \tan \tfrac{1}{2}(A + B) = \frac{VJ}{BJ};$$

$$\frac{\tan \tfrac{1}{2}(A - B)}{\tan \tfrac{1}{2}(A + B)} = \frac{AJ}{VJ} = \frac{AK}{BV} = \frac{a - b}{a + b};$$

or

$$\tan \tfrac{1}{2}(A - B) = \frac{AJ}{BJ} = \frac{AK \sin \tfrac{1}{2}(A + B)}{BV \cos \tfrac{1}{2}(A + B)} = \frac{a - b}{a + b} \tan \tfrac{1}{2}(A + B),$$

(6) Circumscribe a circle about ABC. Draw CP bisecting the angle C. Make $CR = CA = b$. Then $BR = a - b$.



$$\angle CPS = \angle CAR = \tfrac{1}{2}(A + B),$$

$$\angle BPS = \angle BAR = \tfrac{1}{2}(A - B),$$

$$\angle BQS = \angle RQS = \angle BCP = \frac{C}{2},$$

or arc BP = arc PA. Hence

$$\angle RSQ = 90°$$

and

$$BS = SR = \tfrac{1}{2}(a - b),$$

$$CS = \tfrac{1}{2}(a + b).$$

Then we have

$$\tan \tfrac{1}{2}(A - B) = \frac{BS}{PS}; \qquad \tan \tfrac{1}{2}(A + B) = \frac{SC}{PS};$$

$$\frac{\tan \tfrac{1}{2}(A - B)}{\tan \tfrac{1}{2}(A + B)} = \frac{BS}{CS} = \frac{a - b}{a + b}.$$

In addition to the proofs here given I find distinct proofs in R. E. Moritz, *Elements of Plane Trigonometry*, p. 131; Hall and Frink, *Trigonometry*, p. 54; F. Durell, *Plane Trigonometry*, p. 108; in W. F. Cheney, *1920*, 53–54; and in works by the following authors listed *1920*, 53: E. J. Wilczynski, Young and Morgan, and E. W. Hobson.[1]

II. Note on the Law of the Mean.

By H. H. Downing, University of Kentucky.

With certain well-known restrictions on $f(x)$ and its derivative, $f'(x)$, the law of the mean states that

$$\frac{f(x + h) - f(x)}{h} = f'(x + \theta h), \tag{1}$$

where θ is a positive proper fraction.

[1] Yet another proof is given by G. Lindborg in *Tidskrift för Elementär Matematik, Fysik och Kemi*, vol. 3, May, 1920, p. 201.—Editor.

It is desired in this paper to find a relation between θ and h for certain functions. To do this all restrictions on θ and h are removed, allowing them to have any values which are permissible as determined by the functional relation between them. The following cases will be considered:

Case 1. Consider the function $f(x) = ax + b$, $a \neq 0$.

Here $f'(x) = a = f'(x + \theta h)$. $f(x + h) = ax + ah + b$. These values substituted in (1) give the identity $a = a$, from which it is seen that no relation exists between θ and h. This is verified by the geometric interpretation of the tangent to the line whose equation is $y = ax + b$.

Case 2. Consider the function $f(x) = ax^2 + bx + c$, $a \neq 0$.

Obtaining $f'(x + \theta h)$, $f(x + h)$, and substituting in (1) as before it is found that $ah = 2a\theta h$. From this either $h = 0$ or $\theta = 1/2$. The first, $h = 0$, being trivial, geometrically, will not be considered further. The second, $\theta = 1/2$, clearly means that the ordinate of the point of tangency of the tangent which is parallel to the chord joining the points on the parabola $y = ax^2 + bx + c$ with abscissas x and $x + h$ is midway between the ordinates of these points.

Case 3. Next consider $f(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$.

Substitution in (1) gives $ah^2 + bh + 3ahx = 6a\theta hx + 3a\theta^2h^2 + 2b\theta h$. To simplify this equation put $\theta h = \phi$, and the equation then reduces to

$$\frac{\left(h + \frac{3ax + b}{2a}\right)^2}{\left(\frac{3ax + b}{2a\sqrt{3}}\right)^2} - \frac{\left(\phi + \frac{3ax + b}{3a}\right)^2}{\left(\frac{3ax + b}{6a}\right)^2} = -1.$$

This is seen to represent a hyperbola with its principal axis parallel to the ϕ-axis. In this figure θ is the slope of the line from the origin to the point (h, ϕ).

The following results are obtained:

The center is the point $(h, \phi) \equiv \left(-\frac{3ax + b}{2a}, -\frac{3ax + b}{3a}\right)$.

The semi-axes are: transverse $= \frac{3ax + b}{6a}$, conjugate $= \frac{3ax + b}{2a\sqrt{3}}$.

The eccentricity $= 2$.

The foci are: $F \equiv \left(-\frac{3ax + b}{2a}, -\frac{2(3ax + b)}{3a}\right)$ and $F' \equiv \left(-\frac{3ax + b}{2a}, 0\right)$.

The asymptotes are the lines $\phi = \frac{\sqrt{3}}{3}h + \frac{3ax + b}{6a}(\sqrt{3} - 2)$ whose inclination is 30°, and $\phi = -\frac{\sqrt{3}}{3}h - \frac{3ax + b}{6a}(\sqrt{3} + 2)$ whose inclination is 150°.

The vertices are the points

$$V \equiv \left(-\frac{3ax + b}{2a}, -\frac{3ax + b}{2a}\right) \quad \text{and} \quad V' \equiv \left(-\frac{3ax + b}{2a}, -\frac{3ax + b}{6a}\right).$$

It is seen that for F', V', C (the center), V and F, we have

$$\phi = 0, \quad \phi = \tfrac{1}{3}h, \quad \phi = \tfrac{2}{3}h, \quad \phi = h, \quad \phi = \tfrac{4}{3}h.$$

If x increases these points move towards the left along straight lines through the origin, and the hyperbolas corresponding to two values of x are homothetic with respect to the origin.

The interesting point about the equations is the nature of the coefficients of h.

It is further interesting to notice those points on the hyperbola which may be used when the conditions of the law of the mean are satisfied: that is, when θ is positive and less than one, or when ϕ is numerically less than h, and has the same sign. Also, notice may be made of these relations when the point on the cubic whose abscissa is x, is to the left of the inflection point, coincides with the inflection point, or is to the right of it.

RECENT PUBLICATIONS.

REVIEWS.

The Early Mathematical Manuscripts of Leibniz translated from the Latin texts published by Carl Immanuel Gerhardt with critical and historical notes. By J. M. CHILD, Chicago-London, The Open Court Publishing Co., 1920. Price $2.00.

American readers will be glad to possess in English translation the manuscripts of Leibniz containing his early work in the creation of the calculus that were preserved in the Royal Library in Hanover and were first published by C. I. Gerhardt in the years 1846–1855. We have here seventeen manuscripts bearing dates which fall within the time 1673–1677 and also two manuscripts of later date which give two accounts, one brief, the other more extended, of the origin of the calculus—both from the pen of Leibniz himself. Child breaks the chronological order by printing the last two documents first, thus laying stress on the parts which after all are really less important; for it is the papers of 1673–1677 that contain the cardinal points of the record. The book begins with the controversy on the priority of discovery which, by rights, should have been postponed to the end. Child translates also Gerhardt's essay on "Leibniz in London," which includes three manuscripts of Leibniz, and a second essay by Gerhardt, on "Leibniz and Pascal." Child adds profuse historical notes and arguments.

As a rule, the style is clear, but we mention a few exceptions. On page 8, line 23, a sentence is quoted, but it does not clearly appear which of three men mentioned is the author of it; only by inference from what follows later can the reader place the author with certainty. On the same page, line 26, Child quotes from "the memoir referred to," when as yet, no memoir has been mentioned; the authorship of the quotation at the bottom of page 8 is not revealed in the text.

In some instances Child is guilty of inconsistencies and errors. On page 9 he gives 1708, instead of 1704, as the date of the first complete publication of Newton's *Tractatus de quadratura curvarum*. On page 13, line 38, the date of the second edition of Barrow's *Lectiones Opticæ et Geometricæ* is given as 1874; of course, it should be 1674. On page 6 Child declares that Fatio concludes that

Leibniz, as the second discoverer of the calculus, "has borrowed from Newton," while on page 9 Child says, "Fatio does not dare to make a direct assertion, only an insinuation."

Some of the footnotes contain conjectures relating to historical events and the motives which prompted the movements of the actors. These conjectures are of value, provided the rapid reader does not lose sight of the fact that they are only conjectures. Indeed, some of Child's contentions are far from convincing. Take the matter of the "Characteristic triangle." Leibniz used it but does not claim it as his own. He says he got it from Pascal. Child argues that Leibniz must have gotten it from Barrow, on the ground that Leibniz obtained a copy of Barrow's *Lectiones Opticæ et Geometricæ* a month or so before he got a copy of Pascal, and that Leibniz's geometrical figure more closely resembles Barrow's than that of Pascal. Child may be correct in his view that for geometrical knowledge, Leibniz was indebted to Barrow more deeply than is ordinarily believed.

As regards the Newton-Leibniz controversy, Child agrees with A. De Morgan in characterizing the publication of the *Commercium epistolicum* by the Royal Society of London as an act of disgraceful unfairness; Child discards the hypothesis that Leibniz took his calculus from Newton, and says: (p. 5) "I hold quite other views as to the possible source of Leibniz's inspiration, if indeed he is not to be credited with perfectly independent discovery."

There are a few American reviewers who unconsciously have acquired the mannerism of belittling American contributions to the history of science and extolling foreign ones. While in the past, the present reviewer may have exhibited an exactly opposite tendency, he has no hesitation in this case to express the opinion that students of the history of mathematics will feel greatly indebted to Child for his translations and for his conscientious, fair-minded and thorough study of the documents involved.

FLORIAN CAJORI.

UNIVERSITY OF CALIFORNIA.

Infinitesimal Calculus. By F. S. CAREY. (Longmans Mathematical Series.) London, Longmans, 1919. 8vo. 20 + 352 + 9 pages. Price 14 shillings.

How far should one go in introducing new ideas at the beginning of a first course in the calculus? And how much time can one afford to spend in explaining those ideas? It is in answering these two questions that the text before us differs most radically from our standard texts. Most writers seem to agree that while in an introductory course the notions of variable, function and limit must be discussed, a very brief discussion is best, introducing as little notation and as few new terms as possible, with the thought presumably that those notions will become most rapidly and truly understood through use.

The attitude of the author is given in the preface. "Believing that there is no royal road which leads smoothly and directly to the Infinitesimal Calculus, the author has made no attempt to evade all the difficulties which at the outset face the student in this subject. The road has, however, been laid in the first

section so as to pass through those domains of number and function with which the student is probably already acquainted. . . . To assist the student in mastering the fundamental conception of a differential coefficient, two ideas which are usually reserved for books of a more advanced character have been introduced at the beginning and used throughout the book, namely, range and sequence, and the ordinary symbolism in connection with them has been varied. . . . The book . . . is essentially a book of practical mathematics. With this end in view, fundamental ideas are explained at great length; for the easiest and quickest way to master this subject is to acquire a firm grasp of the conceptions upon which it is based. The student is advised to return again and again to the earlier chapters; it is only gradually that the matter contained in them can be assimilated. . . "

In accord with these statements the author has taken more than twice as much time for his introductory material as is common in other introductory texts, and the notions range and sequence are brought into unusual prominence. Most of us will, I believe, doubt the necessity, and hence the desirability also, of directing so much energy toward the mastery of these concepts so early in the student's career. But if we accept the author's point of view, and do not try to quarrel with him over what should be the purpose of his book, we must compliment him upon the skill with which he has carried out his plan. Anyone agreeing with his point of view will be delighted with his full and elementary presentation.

The symbolism referred to for range and sequence is simple and worthy of mention. An open range from a to b is denoted by brackets $[a, b]$, a closed range by parentheses (a, b); and a range open only at one end by the appropriate combination of the bracket and parenthesis symbols; thus a range open at a and closed at b is denoted by $[a, b)$. If a variable x ranges over a sequence having a limit a, one finds the general notation $x \to a$; if the sequence is monotone non-decreasing the notation is $x \rightarrow a$, if monotone non-increasing $x \rightarrow a$. This variation of the arrow notation seems especially simple and suggestive.

The book contains a number of variations from what we are accustomed to, which are interesting. One example is the general use of the symbol D^{-1} for the inverse of differentiation until after a discussion of definite integrals. There will be differences of opinion as to the desirability of delaying the use of the ordinary integral sign in this way. To me it seems best to associate thus D and D^{-1} as operator and inverse, and to have the integral sign presented with the idea of an integral as the limit of a sum. I believe the student will more quickly obtain a correct idea of definite integrals, which is of course a consideration of prime importance, while D^{-1} appears as a more natural notation for the inverse of differentiation.

Another variation from custom is the omission of Duhamel's theorem or any theorem resembling it. Problems which may be solved by the use of simple definite integrals are put into the form of differential equations with initial conditions, without, however, using this terminology. The method is simpler.

I think, than to use Duhamel's theorem, and as logical. It should be added, however, that in the treatment of multiple integrals the author is not so successful, and is forced to disregard small quantities of higher order without explanation.

The American reader is impressed with the amount of geometry that is given. The chapters on curve tracing, envelopes, evolutes, roulettes, and graphics and nomography have much in them that is not included in our introductory texts. As there are no counterbalancing omissions, we would find it necessary to lengthen our course to include it all. The book furnishes an excellent reference work for this material. The last chapter, on "Graphics, nomography," gives in ten pages a good idea of the possibilities of the subject, and should, as the author states, "enable the reader to follow the complicated nomograms which are largely used in France and other countries."

The book as a whole is very carefully written, with surprisingly few errors. An occasional slip is however noted. For example, on page 49, we read that "if $f(x)$ is increasing as x passes through $x = a$, $f'(a)$ is positive," overlooking the possibility of $f'(a) = 0$. Other errors were noted, but are of no great importance.

E. J. MOULTON.

Periodic Orbits. By F. R. MOULTON in collaboration with DANIEL BUCHANAN, THOMAS BUCK, F. L. GRIFFIN, W. R. LONGLEY, W. D. MACMILLAN. (Publication no. 161) Carnegie Institution of Washington, 1920. 4to. 14 + 524 pp.

Quotation from the Introduction: "The problem of three bodies received a great impetus in 1878, when Hill published his celebrated researches upon the lunar theory. His investigations were carried out with practical objects in mind, and comparatively little attention was given to the underlying logic of the processes which he invented. For example, the legitimacy of the use of infinite determinants was assumed, the validity of the solution of infinite systems of non-linear equations was not questioned, and the conditions for the convergence of the infinite series which he used were stated to be quite unknown. These deficiencies in the logic of his work do not detract from the brilliancy and value of his ideas, and his skill in carrying them out excites only the highest admiration.

"The work of Hill was followed in the early nineties by the epoch-making researches of Poincaré, which were published in detail in his *Les Méthodes Nouvelles de la Mécanique Céleste*. Poincaré brought to bear on the problem all the resources of modern analysis. The new methods of treating the difficult problem of three bodies which he invented (*sic*) were so numerous and powerful as to be positively bewildering. They opened so many new fields that a generation will be required for their complete exploration. On the one hand, the results were in the direction of purely theoretical considerations, in which Birkhoff has recently made noteworthy extensions; on the other hand, they foreshadowed somewhat dimly methods which will doubtless be of great importance in practical applications in celestial mechanics. The researches of Poincaré are scarcely less revolutionary in character than were those of Newton when he discovered the law of gravitation and laid the foundations of celestial mechanics.

"In 1896 Sir George Darwin published an extensive paper on the problem of three bodies in *Acta Mathematica*. In mathematical spirit it was similar to the work of Hill; indeed, the methods used were essentially those of Hill, but the problem treated was considerably different. For a ratio of the finite masses of ten to one, Darwin undertook to discover by numerical processes all the periodic orbits of certain types and to follow their changes with varying values of the Jacobian constant of integration. This program was excellently carried out at the cost of a great amount of labor. It gave specific numerical results for many orbits in a particular example.

"The investigations contained in this volume were begun in 1900 and, with the exception of

the last chapter, they were completed by 1912. Those not made by myself were carried out by students who made their doctorates under my direction.

"The following chapters have been heretofore published in substance:"

I. Sections III and IV. *American Journal of Mathematics*, vol. 33 (1911).
II. *Astronomical Journal*, vol. 25 (1907).
III. *Rendiconti Matematico di Palermo*, vol. 32 (1911).
IV. *Transactions of the American Mathematical Society*, vol. 11 (1910).
VII. *Mathematische Annalen*, vol. 73 (1913).
VIII. *Annals of Mathematics*, second series, vol. 12 (1910).
XI–XIV. *Transactions of the American Mathematical Society*, vols. 7 (1906), 13 (1912), 8 (1907), and 9 (1908) respectively.
XV. *Proceedings of the London Mathematical Society*, series 2, vol. 2 (1912).

"The investigations and computations contained in the last chapter were completed in 1917. . . . Most of the computations on which many of the results of the last chapter are based were made by Dr. W. L. Hart and Dr. I. A. Barnett. Without assistance of such a high order the laborious computations could not have been carried out."

Contents: I: "Certain theorems on implicit functions and differential equations" (Moulton and Mac Millan), 1–54; II: "Elliptic motion," 55–66; III: "The spherical pendulum," 67–98; IV: "Periodic orbits about an oblate spheroid" (MacMillan), 99–150; V: "Oscillating satellites about the straight-line equilibrium points (first method)," 151–198; VI: "Oscillating satellites (second method)," 199–216; VII: "Oscillating satellites when the finite masses describe elliptic orbits," 217–284; VIII: "The straight-line solutions of the problem of n bodies," 285–298; IX: "Oscillating satellites near the Lagrangian equilateral triangle points" (Buck), 299–324; X: "Isosceles-triangle solutions of the problem of three bodies" (Buchanan), 325–356; XI: "Periodic orbits of infinitesimal satellites and inferior planets," 357–378; XII: "Periodic orbits of superior planets," 379–388; XIII: "A class of periodic orbits of a particle subject to the attraction of n spheres having prescribed motion" (Longley), 389–424; XIV: "Certain periodic orbits of k finite bodies revolving about a relatively large central mass" (Griffin), 425–456; XV: "Closed orbits of ejection and related periodic orbits," 457–484; XVI: "Synthesis of periodic orbits in the restricted problem of three bodies," 485–524.

Elementary Functions and Applications. By A. S. GALE and C. W. WATKEYS. New York, Henry Holt and Company, 1920. 12mo. 20 + 436 pages. Price $2.60.

From the preface: "This book presents a coherent year's work in mathematics for college freshmen, consisting of a study of the elementary functions, algebraic and transcendental, and their applications to problems arising in various fields of knowledge. The treatment is confined to functions of one variable, with incidental exceptions, and complex values of the independent and dependent variables are excluded. The subject matter includes the essentials of plane trigonometry and topics from advanced algebra, analytic geometry, and calculus.

"The text is the result of experiments beginning in 1907–8. It has been used in the classroom since 1913–14, and each year extensive revisions have been made. Hence the content of the course, the order of topics, and the manner of presentation are based upon the experience of several years.

"The unity of the course is gained by an explicit analysis of the functions studied, which enables the student to comprehend the purpose of the course as a whole and the nature of the investigation of properties of functions of a given type. This analysis consists of three parts:

"First. Relations between a given function and its graph. . . . Most of these relations are considered in the first chapter so that at the start the student is made aware of a number of questions which will be investigated in studying a particular type of functions.

"Second. Relations between pairs of functions and their graphs. . . . These geometric transformations are introduced in connection with simple algebraic functions so that they are familiar tools by the time they are needed for the study of transcendental functions.

"Third. Analogous properties of functions which have no immediate graphical interpretation. Several properties of x^n are grouped together . . . in order to indicate further questions which should be investigated in studying transcendental functions.

"Emphasis is also placed on characteristic properties which distinguish one class of functions from another."

Contents—Chapter I: Functions, equations, and graphs, 1–45; II: Linear functions, 46–86; III: Algebraic functions, 87–157; IV: Trigonometric functions, 158–213; V: Exponential and logarithmic functions, 214–263; VI: Differentiation of algebraic functions, 264–300; VII: Integration, 301–331; VIII: Properties of trigonometric functions (Logarithmic solution of triangles, Cases III and IV), 332–364; IX: Theory of measurement [Statistical methods, permutations, combinations, the binomial expansion, probability, compound events, mortality tables, frequency distributions, averages, measures of variability, equation of the frequency curve representing a symmetrical distribution, the probable error, least squares, correlations], 365–432; Index, 433–436.

Análisis Matemático. Volumen 1: Las nociones fundamentales. By UGO BROGGI. (Publicaciones de la Facultad de Ciencias Fisicas, Matemáticas y Astronomicas, Textos y conferencias no. 44.) La Plata, Imprenta y casa editora 'Coni,' Buenos Aires, 1919. Royal 8vo. 152 p. Price 6 pesos.

Part 1: Coördinates and tangents, 1–46; part 2: The notions of derivative and integral 47–90; part 3: Applications (Elementary methods of integration, definite integrals of geometry indeterminate expressions, integrals between infinite and improper limits, exercises), 91–148; alphabetical index, 149–152.

NOTES.

The first article (pages 33–37) of the second number of the second volume (1920) of *Norsk Mathematisk Tidsskrift* is a paper on "The main lines of H. G. Zeuthen's scientific productivity," read by Professor Heegard before the Norwegian Mathematical Society. It is accompanied by the reproduction of a recent portrait of Zeuthen.

In *The American Year Book, A Record of Events and Progress 1919* (New York, Appleton, 1920), the review of mathematics and astronomy, by G. A. Miller and R. S. Dugan respectively, occupied pages 607–614. The headings of Professor Miller's article (pages 607–608) are: "Teaching of mathematics," "Mathematical research," "History of mathematics," "Reference works."

Mathematics Teacher, in reorganized form, is to be the official organ of the National Council of Teachers of Mathematics (cf. *1920*, 241). Mr. J. R. CLARK of the Lincoln School, New York, is to be the editor-in-chief and the first number is to be issued in January, 1921. There are to be eight numbers a year and the subscription price is $2.00.

In *Archivio di Storia della Scienza*, volume 1, no. 3, published in May, 1920 (cf. *1919*, 404; *1920*, 217) Aldo Mieli continues his methodical bibliography (350 titles) of works on the history of science, 332–356. There is also a review by A. Mieli of two works published by the John Crerar Library, Chicago: *A List of Books on the History of Science, January, 1911* (Chicago, 1911); *Supplement, December, 1916* (Chicago, 1917), 361–363.

A new edition of Professor H. S. Carslaw's book *Introduction to the Theory of Fourier's Series and Integrals and the Mathematical Theory of the Conduction of Heat*, published in 1906 by Macmillan and Co., London, and now for some time out of print, is at present in the press. The book has been completely rewritten and will consist of two volumes. The first deals with "Infinite series and in-

tegrals," with special reference to Fourier's series and integrals; the second with the "Conduction of heat." It is expected that the first volume will appear early next year and the second in the course of that year.

We have received the first fifteen numbers (May, 1919–July, 1920, 12 numbers to a volume) of the Spanish monthly periodical *Revista de Matematicas y Fisicas Elementales*, "designed for secondary, normal, special and higher schools," and published at Buenos Aires. Each number contains short articles (for example on "The indefinite integral," "An artifice in successive approximations," "Trial in classification of cubics," "Logarithmic calculation"), Notes, Miscellanea (a collection of statements of theorems and historical notes), Bibliography (brief reviews, contents of periodicals, titles of new books), Questions and discussions, and Problems proposed and solved—The publication at Buenos Aires of the bimonthly *Revista de Matematicas* ceased in February, 1918, with the concluding number of the second volume.

ARTICLES IN CURRENT PERIODICALS.

AMERICAN JOURNAL OF MATHEMATICS, volume 42, no. 2, April (published in July), 1920: "On the convergence of certain classes of series of functions" by R. D. Carmichael, 77–90; "On the solution of linear equations in infinitely many variables by successive approximations" by J. L. Walsh, 91–96; "Self-dual plane curves of the fourth order" by L. E. Wear, 97–118; "On the groups of isomorphisms of a system of Abelian groups of order p^m and type $(n, 1, 1, \cdots, 2)$" by L. C. Mathewson, 119–128; "On the satellite line of the cubic" by R. M. Winger, 129–135.

BOLLETINO DI BIBLIOGRAFIA E STORIA DELLE SCIENZE MATEMATICHE, second series, volume 2, 1919, no. 1: [Authorized translation into Italian of L. C. Karpinski's article "The History of Science" in *School and Society*, December 21, 1918 [*1919*, 163]], 15–27.—No. 2: Review by G. L[oria] of D. E. Smith's edition of De Morgan's *Budget of Paradoxes* (Chicago, 1915), L. C. Karpinski's *Robert of Chester's Latin Translation of the Algebra of Al-Khowarizmi* (New York, 1915), and of R. C. Archibald's *Euclid's Book on Divisions of Figures* (Cambridge, 1915), 38–41 and 43–44; Review by G. Vivanti of M. Bôcher's *Leçons sur les méthodes de Sturm dans la théorie des équations différentielles* (Paris, 1917) and of H. Hancock's *Theory of Maxima and Minima* (Boston, 1917), 46–49; Brief notes on the following articles by scholars in America: "Pierre Laurent Wantzel," "Plans for a history of mathematics in the nineteenth century," and "Origin of the name 'mathematical induction'" by F. Cajori; "The rôle of the concept of infinity in the work of Lucretius" by C. J. Keyser; "The teaching of the history of science" by G. Sarton; and "Medicine and mathematics in the sixteenth century" by D. E. Smith, 58–64.

BULLETIN OF THE AMERICAN MATHEMATICAL SOCIETY, volume 26, no. 10, July, 1920: "The April meeting of the American Mathematical Society in New York" by F. N. Cole, 433–444; "Stieltjes derivatives" by P. J. Daniell, 444–448; "Sheffer's set of five postulates for Boolean algebras in terms of the operation 'rejection' made completely independent" by J. S. Taylor, 449–454; "Rotation surfaces of constant curvature in space of four dimensions" by C. L. E. Moore, 454–460.

BULLETIN OF THE CALCUTTA MATHEMATICAL SOCIETY, volume 10, no. 4, March, 1920: "On the diffraction of light by a transparent wedge" by S. Banerji, 199–206; "On a geometrical treatment of the scattering of light by a perfectly reflecting cone" by A. Datta, 207–212; "A note on the deformation of surface" by B. Sen, 213–218; "On the stability of two co-axial rectilinear vortices of compressible fluid" by B. Datta, 219–228; "On some properties of natural numbers" by H. Datta, 229–238; "A note on Whittaker's formula for the solution of algebraic or transcendental equations" by B. Pal, 239–242; Review by A. C. Bose of C. E. Cullis's *Matrices and Determinoids* (2 vols., Cambridge, 1913–1918), 243–256.

ENSEIGNEMENT MATHÉMATIQUE, volume 21, no. 1, June 1920: "Comment un conservateur pourait-il arriver au seuil de la mécanique nouvelle?" by T. Levi-Civita, 5–28 (conference at the mathematical seminary of the University of Rome, March 8, 1919); "Remarques sur la théorie

des ensembles et les antinomies cantoriennes, II" by D. Miriamanoff, 29–52; Les deux suites Fibonacciennes fondamentales (u_n) (v_n); tables de leurs termes jusqu'à $n = 120$" by C. A. Laisant, 52–56; "C. A. Laisant (1841–1920)" by H. Fehr, 57; Mélanges, Chronique, 58–63; Review of D. E. Smith's *Number Stories of Long Ago* (Boston, 1919), 67–68.

HERMATHENA, no. 42, 1920: "Heron's formula for cube root" by J. G. Smyly, 64–67; "Some examples of Greek arithmetic" by J. G. Smyly, 105–114.

ISIS, volume 3, January, 1920: "The purpose of Zeno's arguments on motion" by F. Cajori, 7–20.

JOURNAL OF THE INSTITUTE OF ACTUARIES, volume 52, April, 1920: "An elementary demonstration of Stirling's approximate formula for the value of factorial n" by G. J. L., 102–106.

MATHEMATICS TEACHER, volume 12, no. 4, June, 1920: "Some helps and hindrances in teaching mathematics in the secondary schools" by L. E. Lynde, 139–153; "The effect of post-armistice conditions on mathematical courses and methods" by H. English, 154–166; "Teaching practical mathematics efficiently" by C. H. Sampson, 167–171. "New Books," 172–174.

NATURE, volume 105, June 10, 1920: "The approximate evaluation of definite integrals between finite limits" by A. F. Dufton, 455–456—June 17: "A new method for approximate evaluation of definite integrals between finite limits" by T. Y. Baker, 486—July 1: "Use of Sumner lines in navigation" by T. H. Tizard, 552–554; "Dr. F. A. Tarleton" by R. A. P. Rogers, 554; "British aëronautics," 561–562.

PHILOSOPHICAL MAGAZINE, sixth series, volume 40, July, 1920: "A new reading of relativity" by F. Slate, 31–49; "The bearing of rotation on relativity" by R. A. Sampson, 67–72; "On the measurement of time and other magnitudes" by N. R. Campbell, 161–162.

POPULAR ASTRONOMY, volume 28, no. 6, June–July, 1920: "The theory of relativity" by W. H. Pickering, 334–344,—No. 7, August–September: "John Alfred Brashear, 1840–1920" (portrait frontispiece) by F. Schlesinger, 373–379; "Aintoff's equal area projection of the sphere" by R. L. Faris, 385–386.

PROCEEDINGS OF THE AMERICAN ACADEMY OF ARTS AND SCIENCES, volume 55, no. 4, May, 1920: "Orbits from assumed laws of motion" by Arthur Searle, 189–207.—No. 7, June: "Some geometric investigations on the general problem of dynamics" by J. Lipka, 283–322.

PROCEEDINGS OF THE NATIONAL ACADEMY OF SCIENCES, volume 6, no. 5, May, 1920: "On Kummer's memoir of 1857 concerning Fermat's last theorem" by H. S. Vandiver, 266–269.

REVUE DU MOIS, 15th year, April 10, 1920: "Les mathématiques à l'Université de Strasbourg" by M. Fréchet, 337–362.

REVUE GÉNÉRALE DES SCIENCES, volume 31, June 15, 1920: "Un nouvel appareil pour le tracé des coniques," 329–330 (an account of the conigraph as described in the *Proc. Royal Society*, see this MONTHLY, *1920*, 317); "La géométrie differentielle projective des résaux" by G. Tzitzéica, 349–352 [". . . Cependant, c'est à M. Wilczynski que revient le mérite d'avoir créé une théorie projective, aussi complète que possible, des propriétés infinitésimales des courbes planes, des courbes gauches et des surfaces réglées. Il a employé à cet effet certains invariants et covariants des équations différentielles linéaires du troisiéme et du quatriéme ordre et des systèmes d'équations du second ordre. Il a appliqué aussi une méthode analogue à l'étude des surfaces et des congruences de droites de notre espace.

"Cette méthode a donné de beaux résultats. Toutefois les calculs, un peu longs et compliqués, laissent au second plan, les propriétés géométriques. Aussi, les études synthétiques de quelques géomètres italiens out rendu à cette branche de la géomètrie différentielle un aspect plus intuitif, en y ajoutant de plus la considération des figures dans un espace projectif à plus de trois dimensions."]

REVUE SCIENTIFIQUE, volume 58, June 12, 1920: "L'espace et le temps dans la physique moderne" by L. Bloch, 333–341.—June 26: "La théorie d'Einstein et la nouvelle loi de la gravitation" by J. Bosler, 353–359.

SCHOOL AND SOCIETY, volume 12, August 7, 1920: "An odd method for determining the year of birth" by G. A. Miller, 106–107 [De Morgan's conundrum, "I was x years old in the year x^2."]

SCIENCE PROGRESS, volume 15, July, 1920: "Mathematical philosophy" by Bertrand Russell, 101 [Letter dated April 20; "Sir,—The review of my *Introduction to Mathematical Philosophy* by the late P. E. B. Jourdain, in the April number of *Science Progress*, contains some statements which I cannot pass by in silence.

"In the first paragraph, on p. 673, Mr. Jourdain repeats his complaint that I ignored his attempted proof of the multiplicative axiom. In actual fact, I took endless pains over it. At first, I told him where I thought it faulty; he then sent me a new version, altering other parts, but leaving the vital point unchanged. This process occurred repeatedly. I consulted all competent people whom I could get hold of, and they all agreed with me in thinking the proof invalid. I have read his proof with minute care in its various successive forms; I have written to him over and over again explaining where I thought it faulty; but on this point it has not changed. [Cf. this MONTHLY, *1920*, 44.] Mr. Jourdain was a friend of many years' standing, for whom I had both affection and respect, and I was reluctant to enter into public controversy with him. Therefore when I did not think well of his work I did not mention it. Neither in connection with the multiplicative axiom, nor in any other connection, had I any other motive for not referring to him. This applies in particular to the matter mentioned in the second paragraph of p. 673.

"In conclusion, I must entirely repudiate the statement at the top of p. 672: 'Mr. Russell acknowledged the justice of the present writer's strictures in the *Cambridge Review* on a crop of similar errors, but said they were all due to Dr. Whitehead.' This odious accusation must be based upon some conversation of ten years ago, and I can only base my denial upon the certainty that that is not the sort of thing I should do."]; "A psychological geometry" by F. R. Hoare, 131–134 [First two paragraphs: "It is at first sight somewhat strange that the quantitative and statistical methods from which so much was hoped in biology a generation ago should so far have proved more fruitful and suggestive when applied to the apparently much less measurable phenomena of the mind. Yet the hopes that the early biometricians rested on them have certainly not been realised and their present exponents in biology have had to make practically a fresh start with new groups of phenomena and different objectives. In psychology, on the other hand, their recent developments derive in unbroken succession from the work of such pioneers as Binet, who first employed systematic mental tests and measured the results.

"One of the most purely mathematical of these developments was originated by Professor Spearman in a paper published in 1904 in the *American Journal of Psychology*. Its latest and most curious extensions are described in two papers by Mr. Maxwell Garnett published in the *Proceedings of the Royal Society* (A, 96, 1919) and the *British Journal of Psychology* (May 1919) respectively. What follows is largely an account in non-mathematical language of as much of Mr. Garnett's description as can be so expressed."].

SCIENTIA, volume 27, June, 1920: Review by R. Marcolongo of J. W. Young's *I Concetti fondamentali dell' Algebra e della Geometria con una nota di U. G. Mitchell sullo Sviluppo storico del simbolismo algebrico* translated by D. Mercogliano (Naples, 1919), 482–483—July: Review by G. Loria of L. E. Dickson's *History of the Theory of Numbers*, volume 1 (Washington, D. C. 1919), 42–44.

TÔHOKU MATHEMATICAL JOURNAL, volume 17, nos. 3, 4, May, 1920: "Die Zahl: Einleitung zur Theorie der analytischen Funktionen" by G. Mittag-Leffler, 157–209; "On a transformation theorem relating to spheroidal harmonics" by B. Datta, 210–212; "Theorems on double series" by T. Kojima, 213–220; "On a certain inversion in the theory of numbers" by E. T. Bell, 221–231; "On some central difference formulas of interpolation" by K. Ogura, 232–241; "On the theory of the tides" by K. Ogura, 242–250; "A new interpretation of the Wellstein representation and its application," by T. Ota, 251–261; "Some non-euclidean metric theorems as necessary consequences of the corresponding Euclidean theorems" by T. Ota, 262–270; "The ellipsoid within a right circular cylinder" by T. Hayashi, 271–278; "Note on the theory of integral functions of the first class" by R. D. Carmichael, 279–285; "Note on a transformation of series similar to the principle of inversion in the theory of numbers," by R. D. Carmichael, 286–291; "Flat-sphere geometry, fourth paper" by J. Eiesland, 292–330; "On the 'Chinese theorem'" by S. Wigert, 331; "Groups involving only operators whose orders are divisors of four" by G. A. Miller, 332–338; "Ein Satz über die Borelsche Summation" by M. Fujiwara, 339–343; "Ueber Konvergenzabszisse der Dirichletschen Reihen" by M. Fujiwara, 344–350; "Einige Probleme über konvex-geschlossene Kurven und Flächen" by T. Kubota, 351–362; "Ueber Abelsche erzeugende Funktion und Darstellbarkeitsbedingung von Funktionen in Dirichletsche Reihen" by M. Fujiwara, 363–383.

UNTERRICHTSBLÄTTER FÜR MATHEMATIK UND NATURWISSENSCHAFTEN, vol. 26, nos. 3–4, April 20, 1920: "Die Einführung besserer Zahlwörter" by E. Schülke, 30–31; "Zur Lehre von den Harmonikalen" by J. Moser, 31–33; "Ein Polyedersatz" by H. Wolff, 33–34 [The proposition: "If n points $A_1, A_2, \cdots A_n$, the vertices of a polyhedron lie on a sphere in such a way that

its middle point M is the centre of gravity of the A_λ, and if P is any point in space at a distance a from M, then $\sum_\lambda PA_\lambda = n(R^2 + a^2)$"]; "Muss die Sonne ihren höchsten Stand immer im Meridian erreichen"? by P. Kiesling, 34–35; "Die Kugel als regelmässiger Vielflächner" by F. Elsas, 36–37; "Zur Maclaurins Trisektrix, zur Kissoide und zur Versiera" by A. Peschke, 37; "Zur Lehre von den harmonischen Strahlen" by J. Mahrenholz, 37; "Ueber eine Beziehung zwischen Normale und Brennstrahlen in einem Punkte einer Ellipse" by J. Mahrenholz, 37–38; Various other brief notes, 38–40.

ZEITSCHRIFT FÜR MATHEMATISCHEN UND NATURWISSENSCHAFTLICHEN UNTERRICHT, volume 51, no. 4–5, published May 20, 1920: "Das d'Hondtsche Wahlsystem." "Eine Anwendung der graphischen Methoden" by E. Sós, 89–93; "Mathematische Betrachtungen über das geltende politische Wahlverfahren" by H. Franke, 94–106; "Arbeit und Boden in der Volkswirtschaftslehre und Mechanik" by J. Braun, 106–109; "Optische Geometrie" by R. Böger, 110–118; "Eine neue Verallgemeinerung des pythagoräischen Lehrsatzes" by A. Voigt, 118–122; "Das Ecktangentendreieck" by J. Mahrenholz, 123; "Der Saccheri-Legendresche Satz lautet: Ist in einem einzigen Dreieck die Winkelsumme $\leqq 180°$, so ist sie es in jedem" by K. Fladt, 124–125 [a very simple proof for the Euclidean case, which, however, the author states he has not yet succeeded in extending to the non-Euclidean cases;] "Aufgabenrepertorium," 126–132; "Bücher, besprechungen," 140–141.

UNDERGRADUATE MATHEMATICS CLUBS.

EDITED BY U. G. MITCHELL, University of Kansas, Lawrence.

Last month we gave some account of two new clubs, the "Pascal Circle" of Trinity College, Washington, D. C., and "The Square" of Washington Square College, New York City, which had not been previously reported. This month our readers will note among the "Club Activities" an account of a third new club, "The Cornell Parabola," organized in November 1919 at Cornell University, Ithaca, N. Y.

The editor of this department has recently enjoyed the privilege of examining a copy of the "*Proceedings of the Johns Hopkins University Mathematical Club*, edited by G. Breit and F. V. Morley and printed by Corona Morley."[1]

It is a typewritten volume of about sixty pages, bound in manila paper covers, and reports the nine meetings held in March, April and May, 1920. Many of the formal papers and solutions of problems presented are recorded in full. Others are given in abstract form only. The editors have added to the interest and value of the record by inserting comments and "gleanings" from mathematical literature in connection with the subjects discussed. As an illustration of these "gleanings" we select the following, inserted (p. 34) in connection with a paper on "Quaternions" by Professor Murnaghan:

"'Comparing a Quaternion investigation, no matter in what department, with the equivalent Cartesian one, even when the latter has availed itself to the utmost of the improvements suggested by Higher Algebra, one can hardly

[1] "Corona Morley" refers, as the reader doubtless recognizes, to Mr. Morley's typewriter. The humorous touch added by this bit of personification is supported throughout by occasional references to Corona as, for example, the following (p. 12):

"No account of Mathematical Prodigies will hereafter be complete without reference to Corona Morley. Barely a week old, she is nevertheless able to talk about the Gamma function. The editors wish the best of success to her acquisitive and expository powers."

help making the remark that they contrast even more strongly than the decimal notation with the binary scale, or with the old Greek arithmetic—or than the well-ordered subdivisions of the metrical system with the preposterous no-systems of Great Britain, a mere fragment of which (in the form of Table of Weights and Measures) form, perhaps the most effective, if not the most ingenious, of the many instruments of torture employed in our elementary teaching.'—P. G. TAIT. (Quoted by MORITZ, *Memorabilia Mathematica*, p. 280.)

'Quaternions came from Hamilton after his really good work had been done; and though beautifully ingenious have been an unmixed evil to those who have touched them in any way, including Clerk Maxwell.'—WILLIAM THOMSON. (Quoted by MORITZ, *Memorabilia Mathematica*, p. 279.)

So T and T′ disagree
On parts of their philosophy."

It may be possible that other clubs are keeping similarly complete records of their proceedings. If so, the editor would be glad to have the fact reported.

CLUB ACTIVITIES.

THE CORNELL PARABOLA, Cornell University, Ithaca, N. Y.

The Cornell Parabola was organized by the students of the upper classes in November, 1919. It was modelled along the lines of the Oliver Mathematical Club, which has existed among the faculty members for many years, and the organization was very informal. There is no constitution, there are no dues and there are only two officers—a chairman, popularly known as the "Focus" and a secretary, better known as the "Directrix." A program committee consisting of four students and one faculty member planned the work.

The attendance for all of the meetings averaged over fifteen and there were always spirited discussions and incidental problems at the close of each program.

The only social event of the year was the Christmas party which was held at the home of one of the professors. The chief game of the evening was a contest in supplying mathematical terms for the missing words in a "Romance of Polly Hedron and Ray Show." There was a Christmas tree loaded with candy boxes in all the regular polyhedron shapes.

The officers of the club were: Chairman, Theodore L. Bennett '21; secretary, Elfrida Heath '20; chairman program committee, Nellie G. Tallman '20; faculty adviser, Dr. Helen B. Owens.

Programs for the year 1919–20 are given below.

"Reliability of averages" by Herbert Sturges, Gr; "Properties of determinants" by Theodore L. Bennett '21; "The ancients' knowledge of mathematics" by Elfrida G. Heath '20; "Some applications of mathematics to physics" by Roy Kennedy '21; "College students' errors in mathematics" by Alan D. Campbell, Instructor in Mathematics; "Fallacious proofs in geometry" by Professor Frederick W. Owens; Parabola Christmas Party; "Possible constructions with straight edge and compass" by Professor Owens; "Topics from

mechanics and hydraulics" by Salvador Quinones '21; "Famous problems of mathematics" by L. Kalles '19; "Consequences of a triangular unit of area" by Professor Walter B. Carver; "Properties of the parabola" by Allen D. Campbell, Instructor in Mathematics; "The theory of the pendulum" by Joseph A. Beeker '19; "Research problem in electricity" by Ruth Yeaton, Gr; "Value of mathematics to the engineer" by Burdette K. Northrup '19; "Value of mathematics to the physicist and chemist" by Austin Bailey, Gr.

THE MATHEMATICAL CLUB, Harvard University, Cambridge, Mass.
[*1918*, 186–7, 449–50; *1919*, 262–3.]

The officers for 1919–20 were: President, Joseph L. Walsh, Gr.; secretary-treasurer, Harold T. Davis, Gr.; faculty adviser, Professor William C. Graustein.

Programs for 1919–20 are given below.

October 1, 1919: "The elementary functions" by Professor William F. Osgood.

October 15: "Problems connected with submarine acoustics" by Professor Oliver D. Kellogg.

October 29: "Elementary notions of general analysis" by Dr. Israel A. Barnett, Instructor.

November 12: "Correlation" by Professor Edward V. Huntington.

December 3: "Linkages" by Bancroft H. Brown, Instructor.

December 17: "Nomography" by Rexford S. Tucker, Gr.

January 7, 1920: "Numerical integration" by Charles A. Rupp, Instructor.

February 11: "The application of the method of least squares to some geometrical problems" by Professor Julian L. Coolidge.

February 25: "Operations generating algebra" by Dr. Norbert Wiener, Instructor in Mathematics, Massachusetts Institute of Technology.

March 10: "Approximation to continuous functions" by Lewis E. Ward, Gr.

March 24: "Rectifiable and other curves" by Professor Edward B. Van Vleck, Professor of Mathematics, University of Wisconsin.

April 7: "Polygons whose vertices lie on a convex curve" by Forrest H. Murray, Gr.

April 28: "Surfaces of negative curvature" by Dr. Harold C. M. Morse, Instructor.

May 12: "A solution of the biquadratic" by Professor William C. Graustein.

May 24: "The location of the roots of the derivative of a polynomial" by Joseph L. Walsh, Gr.

Officers-elect for the year 1920–21: President, Rudolph E. Langer, Gr.; secretary-treasurer, Bancroft H. Brown, Instructor; faculty adviser, Professor Oliver D. Kellogg.

THE MATHEMATICAL CLUB OF SMITH COLLEGE, Northampton, Mass.
[*1918*, 91, 455; *1920*, 184.]

Below are given the programs for the year 1919–20.

October 20, 1919: Discussion of plans for the year. Reports of the September

meetings of the American Mathematical Society and the Mathematical Association of America.

November 10: "Map drawing, with special reference to the use of maps in the Great War" by Professor Suzan R. Benedict.

December 1: "Leibnitz" by Ellen Hastings '20; "Descartes" by Marjorie Lord '20; "Newton" by Helen Frank '20.

December 15: Christmas Party.

February 19, 1920: "Applications of mathematics" by Margaret Doran '20.

March 8: "Introductory paper concerning the three famous problems of antiquity" by Agnes Grant '20; "Trisection of the angle" by Isabel Painter '20; "Squaring the circle" by Miriam Courtney '20.

April 12: "Duplication of the cube" by Carolyn Boudo '20; "The Pythagorean theorem" by Ruth Colsten '20.

May 3: "A study of transformations" by Dorothy Graves '21.

May 24: Spring Party.

The Pascal Circle, Trinity College, Washington, D. C.

[*1920*, 425.]

An account of this club's activities for the year 1919–20 was given last month. No previous account of the club having appeared in this department, an inquiry was addressed to Professor Marie Cecelia Mangold concerning the previous history of the club. The information given below was received in reply.

The Pascal Circle was founded in October 1916 with the following officers: President, Anna Marie Lawler '17; secretary, Zita Louise Donahoe '17.

Two meetings per month were held and some of the topics discussed were the following: The mathematical basis of magic number cards and tricks with cards—Geometrical fallacies—Popular "catch" problems—Magic squares and magic pencils—The arcascope and the slide rule—Geometric paper folding—The Pascal triangle and the Pascal theorem—Nine-point circle.

Officers for the year 1917–18 were: President, Virginia Alma Keller '18; secretary, Frances Dix Wyman '19. Some meetings were held in the fall of 1917, but on account of sickness the Circle was disbanded until late in the year 1918–19 when it was reorganized with Katherine Mary Martin '19 as president and Beatrice Agnes Convey '20 as secretary. Three of the subjects discussed that year were: Locating machine guns by means of sound—The mathematical principle underlying the Russian peasant method of multiplication—Ciphers and cryptograms.

PROBLEMS AND SOLUTIONS.

EDITED BY B. F. FINKEL AND OTTO DUNKEL.

Send all communications about Problems and Solutions to B. F. FINKEL, Springfield, Mo.

PROBLEMS FOR SOLUTION.

[N.B. The editorial work of this department would be greatly facilitated if, on sending in problems, the proposers would also enclose their solutions—*when they have them.* If a problem proposed is not original the proposer is requested *invariably* to state the fact and to give an exact reference to the source.]

2863. Proposed by A. A. BENNETT, Baltimore, Md.

From their generation as roulette curves, show that the two hypocycloids of five cusps drawn with common vertices, are such that each is the envelope of a chord of constant length suitably placed upon the other.

Show that for any odd prime p, the $(p-1)/2$ distinct p-cusped hypocycloids with common vertices may be arranged in cycles, so that each is the envelope of a chord of constant length taken upon the succeeding curve of the cycle.

2864. Proposed by C. B. HALDEMAN, Ross, O.

If S is a side of a regular undecagon inscribed in a circle of radius unity, show that

$$S^5 - (S^4 - 3S^2 - 1)\sqrt{11} - 11S = 0.$$

2865. Proposed by JOSEPH ROSENBAUM, Milford, Conn.

In a circle, a chord AB is fixed in position and a moving chord CD is constant in length. Find the locus of the intersection of the bisectors of the angles ACD and BDC.

2866. Proposed by NORMAN ANNING, University of Michigan.

Equilateral triangles whose sides are 1, 3, 5, 7, $\cdots$ are placed so that their bases lie corner to corner in a straight line. Show that the vertices lie upon a parabola and are all at integral distances from its focus.

2867. Proposed by W. H. HAYS, Colton, Wash.

Show that the probable value of the sum of the squared differences, got by pairing two sets of n numbers each, at random, is $n(n^2-1)/6$; and that the coefficient of correlation in such a case is 0. See Thorndike, *Mental and Social Measurements*, page 164.

2868. Proposed by H. S. UHLER, Yale University.

Let the evolute of a given curve be called the evolute of the first order, let the evolute of the first evolute be called the evolute of the second order, etc. Then, being given the following parametric equations in which a is a constant and γ is the parameter, namely,

$$x = (1 + 2\sin^2\gamma)\cos\gamma - a\sin 2\gamma,$$

$$y = 2\sin^3\gamma + a\cos 2\gamma$$

find: (a) the parametric equations of the evolute of order n, both for n even and for n odd;
(b) a formula for the total length of the nth evolute;
(c) a formula for the total area of the nth evolute;
(d) the sum of the lengths of all the evolutes from; $n = 1$ to $n = \infty$; and
(e) the sum of the areas of all the evolutes from $n = 1$ to $n = \infty$.

Note: The original equations represent the envelope required in problem 2819, *1920*, 134.

SOLUTIONS OF PROBLEMS

2790 [1919, 414]. Proposed by J. W. LASLEY, JR., University of North Carolina.

How shall we buy twelve eggs for eighty cents, if hen eggs sell at five cents each, duck eggs at seven cents each, and turkey eggs at eight cents each, and if we buy some of each?

SOLUTION BY C. A. ISAACS, State College of Washington.

Let x, y, z, be the number of eggs of each kind, respectively. Then

$$x + y + z = 12,$$

$$5x + 7y + 8z = 80.$$

Eliminating x,

$$2y + 3z = 20, \quad \text{or} \quad y = 10 - \frac{3z}{2}.$$

Since y is an integer, this value of y, though fractional in form, must be an integer. Therefore, z is even. Assigning even values to z, beginning with 2, we get three sets of answers satisfying the conditions of the problem: $z = 2, y = 7, x = 3$; $z = 4, y = 4, x = 4$; $z = 6, y = 1, x = 5$.

Also solved by H. N. CARLETON, H. L. OLSON, ARTHUR PELLETIER, and E. E. WHITFORD.

NOTES AND NEWS

It is hoped that readers of the MONTHLY will cooperate in contributing to the general interest of this department by sending items to the Editor-in-Chief.

Miss MARY C. BALL has been appointed instructor in mathematics at Northwestern University.

Assistant Professor J. N. MICHIE, of the Agricultural and Mechanical College of Texas, has been appointed adjunct professor of applied mathematics at the University of Texas.

Assistant Professor A. C. MADDOX, of the Oklahoma Agricultural and Mechanical College, has been appointed professor of mathematics at the Louisiana State Normal School at Natchitoches.

Professor J. B. FAUGHT of the Kent (Ohio) State Normal College has been made professor of mathematics in Yankton College.

Dr. G. W. SMITH, of the University of Kentucky, has been appointed assistant professor of mathematics in the University of Kansas. He taught during the past summer in the summer session of the University of Colorado.

Professor R. A. WELLS, of Park College, has been made associate professor of mathematics at Michigan State Normal College.

Mr. CORNELIUS GOUWENS, of the University of Kansas, has been appointed assistant professor of mathematics at Iowa State College.

Professor C. E. HORNE, of the University of Porto Rico, has been made dean of the college of agriculture and mechanic arts of the university at Mayagüez, P. R.

At the University of Iowa, Assistant Professor E. W. CHITTENDEN has been promoted to an associate professorship; Dr. W. H. WILSON has been promoted from an instructorship to an associateship; Dr. ROSCOE WOODS, of the University of Illinois, has been appointed instructor in mathematics; Mr. H. M. JEFFERS of the Lick Observatory has been appointed instructor in mathematics and astron-

omy; Mr. E. S. HARPER, of Albion College, Mr. HARLEY CHANDLER, of Coe College, and Miss MARGARET WALKER, of the University of Illinois, have been appointed assistants in mathematics.

Dr. A. S. HATHAWAY, since 1891 professor of mathematics at the Rose Polytechnic Institute, has retired from active service. He is succeeded by Associate Professor C. P. SOUSLEY of Pennsylvania State College.

Professor H. E. BUCHANAN, of the University of Tennessee, has resigned to become professor and head of the department of mathematics at Tulane University.

Professor E. W. BROWN, of Yale, is on leave of absence for the first semester of this year, a part of the time being spent at Christ's College, Cambridge.

Mr. H. A. SIMMONS has been appointed instructor in mathematics at the University of Michigan.

Dr. HENRY W. STAGER, for many years head of the department of mathematics in Fresno Junior College, Fresno, California, and more recently with the United States Railroad Administration, has been appointed instructor of mathematics in the University of Washington.

At the United States Naval Academy, Assistant Professors J. A. BULLARD, J. N. GALLOWAY, A. DILLINGHAM, and G. R. CLEMENTS have been promoted to associate professorships; Mr. H. M. ROBERT, Jr., Mr. M. A. EASON, Dr. L. S. DEDERICK, Dr. L. T. WILSON, Mr. H. H. GAVER, and Dr. W. F. SHENTON have been promoted to assistant professorships, and Mr. E. R. C. MILES, Mr. A. J. BARRETT, Mr. A. A. ROBINSON, and Mr. E. A. BAILEY have been appointed to instructorships. There are now forty men in the mathematics department at the Academy. All grades have received increases in salary during the past year, instructors now being paid $2,800 per annum.

Sir JOSEPH LARMOR, of the University of Cambridge, has been elected "correspondant" in place of Professor Liapounoff (cf. *1920*, 179, 384) in the section of geometry of the Academy of Sciences of the Institute of France.

Dr. PIERRE BOUTROUX, professor of differential and integral calculus at the University of Poitiers, and recently of Princeton University, has been appointed professor of the history of science at the Collège de France.

Dr. LUDWIK SILBERSTEIN, well known on this side of the Atlantic for his books on the *Theory of Relativity* (1914) and *Elements of Vector Algebra* (1919), and for his mathematical papers dealing with electromagnetism, optics, projective geometry, spectrum theory, etc., has left England and is now associated with the research laboratory staff of the Eastman Kodak Company (see *1920*, 217).

At the thirteenth regular meeting of the Association of Mathematics Teachers of New Jersey, at Rutgers College on October 30, the following papers were read by members of the Association: "The report of the National Committee from the viewpoint of college entrance requirements" by Dean H. E. HAWKES of Columbia University; "The law of exponents" by Professor RICHARD MORRIS of Rutgers College.

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ERRATA AND CORRIGENDA.

P. 41, l. 22, *for* "Simson" *read* "Simpson."
P. 41, l. 29, *for* "DANIEL" *read* "DANIELL."
P. 45, ll. 13–14, *for* "Journal of the Mathematical Association of Japan for Secondary Schools" *read* "*Journal of the Mathematical Association of Japan for Secondary Education.*"
P. 46, l. 3, *for* "*Whitaker*" *read* "*Whittaker.*"
P. 72, ll. 25–26, *for* "H. PIOGGIO" *read* "H. T. H. PIAGGIO."
P. 80, l. 5, *for* "**2806**" *read* "**2806**A."
P. 92, l. 4 from bottom, *for* "THORNDYKE" *read* "THORNDIKE."
P. 192, l. 12, *for* "June 1" *read* "June 21."
P. 199, l. 7, *for* "meets" *read* "meet."
P. 221, l. 25, *for* "J. C. Camplain" *read* "J. C. Kamplain."
P. 221, l. 28, *for* "E. R. Brestich" *read* "E. R. Breslich."
P. 240, l. 24, *for* "E. T." *read* "E. J."
P. 241, l. 9 from bottom, *for* "Columbus" *read* "Calculus."
P. 281, ll. 7–8 from bottom, *for* "America, and one member each by the National" *read* "America and its representative on the National Research Council, and one member each selected by the National."
P. 286, l. 3, *for* "the catenary" *read* "the equation of the catenary."
P. 286, l. 5 from bottom, *for* "then" *read* "and."
P. 288, ll. 26–27, *for* "experimenters" *read* "experiments."
P. 321, l. 6 from bottom, *for* "Burman" *read* "Burnam."
P. 336, l. 10, *for* "associate professor of mathematics" *read* "associate in mathematics."
P. 336, l. 13, *for* "J. P. MUSSELMAN" *read* "J. R. MUSSELMAN."
P. 338, l. 18, *for* "Stackel" *read* "Stäckel."
P. 339, *for* ll. 6–7 *read* "l'Avancement des Sciences. He wrote: *La Mathématique. Philosophie. Enseignement* (Paris, 1898: second edition revised, 1907) and *Initiation Mathématique. Ouvrage étranger à tout programme, dedié aux amis de l'enfance* (Paris, 1906) which was translated into."
P. 339, l. 21, *for* "variants" *read* "invariants."
P. 340, l. 5 from bottom, *for* "1920" *read* "**1920**."
P. 341, l. 11, *for* "1903" *read* "1901."
P. 346, ll. 6–7, *for* "Julia Calpitts" *read* "Julia T. Colpitts."
P. 384, l. 8 from bottom, *for* "25–803" *read* "25 + 803."
P. 385, l. 6 from bottom, *for* "S. H. MACDONALD" *read* "S. L. MACDONALD."
P. 389, l. 5 from bottom, *for* "PAIGE" *read* "PAGE."
P. 428, l. 10 from bottom, *for* "frustrum" *read* "frustum."
P. 481, l. 13, *for* "Carolyn Bonds" *read* "Carolyn A. Boudo."

"Thou hast made me known to friends whom I knew not. Thou hast given me seats in homes not my own. Thou hast brought the distant near and made me a brother of the stranger."

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By MILDRED E. CARLEN, Brown University.

Misprinted names in the text are corrected, and missing initials supplied, in this index. The authors of certain anonymous contributions are also indicated.

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